A Model of Entrepreneurial Clusters, with an Application to the Efficiency of Entrepreneurship

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Abstract

I model an entrepreneurial cluster as three interconnected kinds of markets: the market for venture capital, the labor market for high-skilled individuals, or "engineers," and the product markets startups hope to serve. Engineers choose to either found a startup as an entrepreneur—pursuing a single business idea and obtaining seed funding from VCs in exchange for equity—or join an established startup as an employee. The model predicts the equilibrium number of startups formed and their success probability, as well as the wages of engineers, the share of equity retained by founders, the fraction of engineers pursuing entrepreneurship, and the profits of successful startups. The equilibrium is affected by the cost of founding a startup, the extent of product markets, the supply of engineers, and the "supply" of startup ideas. The model predicts the total output of the entrepreneurial system and the social value of the marginal startup.

1 Introduction

Would-be entrepreneurs must decide not only whether to enter into entrepreneurship, but also which business idea to pursue with their startups. For the entry decision, they presumably

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compare the expected returns from entrepreneurship to their next best option—typically wage employment. For the decision about which idea to pursue, entrepreneurs must consider the size the product market associated with that idea (and the risk that there is not actually any demand for their product), as well as the technical risk that they will not be able to create the product. They also must consider the risk posed by competition from other entrepreneurs pursuing that same product market, with more or less the same business idea.

If "idea competition" were rare, perhaps entrepreneurs would view it as a secondary consideration, but it is easy to point to collections of startups all pursuing more or less the same business idea.[1] It seems implausible that savvy would-be entrepreneurs would not consider the competition they are likely to face. The threat of competition from other entrepreneurs makes the idea selection problem a strategic one. Although entrepreneurs would, all else equal, like to pursue "better" ideas, similarly situated entrepreneurs have the same incentive, and so startups pursuing "good" ideas must expect more competitors. Illustrating the point, Paul Graham, the founder of startup incubator Y Combinator, writes "the best startup ideas seem at first like bad ideas. I've written about this before: if a good idea were obviously good, someone else would already have done it. So the most successful founders tend to work on ideas that few beside them realize are good."[2]

This equilibrium considerations needed to understand entrepreneurial decision-making goes beyond idea selection. For one, in entrepreneurial clusters such as Silicon Valley, would-be entrepreneurs are drawn from the same pool of highly skilled individuals needed to "scale" successful startups as employees. Because of this common labor pool, the returns to the non-entrepreneurial outside option of employment depend not just on the supply of engineer employees—which is affected by the fraction of engineers choosing not pursue entrepreneurship—but also

[1] As a case in point, as of October 1st, 2016 there are no fewer than four different venture-backed "gas on demand" startups. Each has the same idea of fueling up cars by bringing a tanker truck to customers who schedule with a smartphone app—they are WeFuel, FuelMe, Filld, and BoosterFuel.

[2] "Black Swan Farming", http://www.paulgraham.com/swan.html.

the demand for engineers created by successful startups, which are the creation of those engineers that did chose entrepreneurship. Fairlie and Chatterji (2013) demonstrate this connection empirically, showing that a tight Silicon Valley labor market reduced the formation of new startups.[3] The wages paid to engineers also affects the profitability of successful ventures, which in turn also bears on the entrepreneurial entry decision. In short, there are several interconnected markets that must be considered all at once to understand the equilibrium entrepreneurial "system," or cluster.

In this paper, I present an equilibrium model of innovative entrepreneurial clusters as three interconnected markets: the market for venture capital, the labor market for startup employees, and the product markets for what successful startups sell. The key economic actors in the model are "engineers"—those individuals with the human capital required to participate in innovative entrepreneurship, as either startup employees or entrepreneurs. The model's key innovation is the explicit modeling of idea competition, which when added to the basic "occupational" model of entrepreneurship yields a number of new insights, particularly about the efficiency of the entrepreneurial system.

In the model, engineers choosing to be entrepreneurs obtain seed capital by selling equity in their startups to VCs. Each funded startup pursues a single business idea, with ideas differing in their perceived "quality," which is common knowledge. The quality of an idea is the probability it will lead to a "viable" product market if pursued by at least one startup. If the idea proves to be viable, one—and only one—of the startups pursuing that idea wins the product market, earning a profit. The successful startups demand engineer labor; engineers that chose not to pursue entrepreneurship supply it. The wages that engineers can earn in the labor market is the opportunity cost of entrepreneurship.

[3] In other industries, this would be a puzzling finding, as the tight labor market, absent some negative labor supply shock, would indicate rising product market demand for what firms in that industry sell, which should induce more entrepreneurship. The finding is not puzzling when we recognize that those would-be entrepreneurs were having their wages bid up in the labor market.

In the model, the returns to both entrepreneurship and employment are declining in the number of entrepreneurs that pursue that occupation. Employment returns decline because more supply lowers a worker's marginal product, while entrepreneurship returns decline because the startup success probability declines with more competition from other entrepreneurs. In equilibrium, the returns to employment and entrepreneurship are the same, making engineers indifferent between occupations. Entrepreneurs are also indifferent over the business ideas being pursued, as "good" ideas get more entrants. Similarly, VCs are indifferent over all funded startups, as they all offer the same expected return.

The model predicts the wage of employees, the equity retained by entrepreneurs, the fraction of engineers engaged in entrepreneurship, the number of startups formed, the startup probability of success, the size of successful startups (the number of employees), expected startup profits (and hence early-stage startup valuations), and realized profits for successful startups. This equilibrium is affected by exogenous factors, namely the direct cost of founding a startup, the extent of product markets, the supply of engineers, and the "supply" of business ideas. When a shock occurs to some exogenous factor, it alters the relative returns to employment and entrepreneurship. To restore an equilibrium, engineers flow from the relatively disadvantaged occupation to the relatively advantaged occupation.

Overview of the comparative statics. A decrease in startup costs is good for engineers: it raises their wages if they are employees, and it raises their retained equity if they are entrepreneurs. With lower startup costs, more engineers shift to entrepreneurship, increasing idea competition and lowering startup success probabilities. However, more ideas are pursued overall, and so the number of successful startups increases. This increase in the total number of successful firms is why wages increase—even though engineers are paid their marginal revenue product, with more startups, there are fewer engineer employees per firm, making the marginal employee more productive.

If more engineers are added to the system, wages and retained equity fall. Profits increase because of lower labor costs, but expected profits fall because the influx of engineers lowers the startup success probability enough to offset the increase in realized profits. The split of new engineers into either entrepreneurship or employment does not necessarily follow the pre-shock split: a shock can be either entrepreneur-biased or employment-biased, depending on the elasticity of labor demand for engineers. When the labor market can readily absorb new engineers with little decrease in wages, then new entrants are biased towards employment, and vice versa when demand is inelastic.

Before engineers can change occupations, an expansion of the product market increases both the profits of successful startups and the wages of their employees. When the product market expands, but before engineers can change occupations, wages and profits rise by the same percentage. This would seemingly keep the relative attractiveness of employment and entrepreneurship unchanged. However, the increased profits allow entrepreneurs to obtain better terms from VCs, making entrepreneurship relatively more attractive. As such, the net effect of an expanded product market is higher wages, more entrepreneurship, and a lower startup success probability.

The net effect of an increase in the number of ideas is higher wages for employees and a higher startup success probability. However, it is ambiguous whether entrepreneurship increases. With more ideas, there is initially less crowding among entrepreneurs, and so the startup success probability increases, which increases the number of successful, labor-demanding startups. However, with higher demand, wages rise, and so which occupation is relatively more advantaged by the shock depends on the elasticity of labor demand. After engineers can change occupations, expected profits increase, but realized profits fall because of the higher labor costs.

Overview of the efficiency results. A key focus of the analysis is on the efficiency of the entrepreneurial system. Kihlstrom and Laffont (1979), which present an occupational model with a similar setup, find that the amount of entrepreneurship is efficient, so long as entrepreneurs are risk neutral. Despite a similar basic setup, I find that there is too much entrepreneurship. The reason for the difference is that in my model, entering entrepreneurs have a business stealing effect on their competitors because of idea competition, and as in Mankiw and Whinston (1986), would-be entrants do not internalize this externality. The same could seemingly be said about entrants into employment that also have a pecuniary externality on other employees, but the economic difference is that entrepreneurship has a fixed cost that employment lacks. Note that either feature in isolation—(1) business stealing but no entry cost, or (2) entry cost without business stealing—would lead to an efficient amount of entrepreneurship.

In my model, the decentralized equilibrium is efficient only when the marginal engineer moving to entrepreneurship pursues an idea that was not already being pursued. While the probability of duplicative entry might be difficult to measure in practice, I show that it is equal to the elasticity of the startup success probability with respect to the number of entrepreneurs—a parameter that might be easier to identify credibly.

One way to reduce the inefficiency caused by excess entry in to entrepreneurship is to make the startup success probability less elastic with respect to the number of entrepreneurs. One attractive way to do this, in terms of the model, is to make more ideas available, such as through public R&D spending, which reduces entrepreneurial crowding. This is, as far as I am aware, a novel market failure argument for the desirability of public R&D spending, distinct from the more common "public good" argument.

The net output of the entrepreneurial system—the total revenue product minus the total founding cost of all startups—is simply the total number of engineers times the market wage. The engineer wage turns out to be a "sufficient statistic" for total output.[4] This is useful, as it makes assessing the efficiency implications of various policy interventions straightforward. The model offers comparative statics on factors that are strongly influenced by public policy, such

[4]See Chetty (2009) for details.

as the supply of ideas and the supply of engineers. These comparative static results are not just directional, but rather have magnitudes defined in terms of more fundamental parameters. For example, the elasticity of labor demand is particularly important, as the magnitude determines the direction of some comparative static results, such as whether an expanded product market would increase or decrease entrepreneurship. This feature of the model is attractive given the policy interest in stimulating entrepreneurial clusters and the fairly scant guidance on what policies are likely to be effective or how their impacts should be measured (Chatterji et al., 2014).

In the model, a single startup is sufficient to realize the full social benefit of some idea. Of course, startups fail for idiosyncratic reasons, and there are certainly other benefits to idea competition, such as faster innovation. A social planner might want multiple startups per idea, or more entrepreneurs pursuing relatively better ideas. The decentralized allocation of entrepreneurs does have more than one entrepreneur per idea, and relatively better ideas get more entrepreneurs. However, under the assumption that each pursuing startup has some independent probability of failure, I show that the decentralized equilibrium has too much entry on the "best" ideas.

Structure of the paper. Section 2 explores what is distinctive about technology entrepreneurship and reviews extant models of entrepreneurship. Section 3 develops the model and then presents the comparative static results. Section 4 explores the social efficiency of the equilibrium and how different assumptions about idiosyncratic risks of failure would affect the efficiency analysis. Section 5 concludes with some thoughts on directions for future research.

2 Related work

Venture-backed, growth-oriented innovative entrepreneurship, despite its importance, is a small fraction of all entrepreneurship: Åstebro et al. (2014) write that "venture-capital-backed startups account for under 1 percent of the startups founded each year and typically focus on higher-growth ventures commercializing new technologies or products." This rarity matters, as it is likely that some of what we know about entrepreneurship generally does not necessarily apply to innovative entrepreneurship. In this section, I highlight differences between innovative and conventional entrepreneurship and explain how these differences are reflected in the assumptions of the model.

Conventional entrepreneurs enter established markets, intending to sell the same product, more or less, as incumbents. These ventures have limited potential upside, and as such, conventional entrepreneurs are often self-financed. Herranz et al. (2013) show that small business entrepreneurs have substantial personal wealth invested in their firms.[5] Relatedly, the literature shows that entry into entrepreneurship is positively correlated with wealth (Evans and Jovanovic, 1989; Blanchflower and Oswald, 1998; Holtz-Eakin et al., 1994; Lindh and Ohlsson, 1996; Nanda, 2010).[6] Presumably part of the reason for this relationship is that wealthier individuals are better able to bear the risk of entrepreneurship.

The relationship between entrepreneurial risk and reward is the core of the Knight (1921) view of entrepreneurship. Kihlstrom and Laffont (1979) formalize this view, presenting a general equilibrium theory of firm formation. The risk-based view is not the only view of entrepreneurship. The competing view is that entrepreneurs see business opportunities missed by others (Schumpeter, 1947; Schultz, 1975; Rosen, 1983; Holmes and Schmitz Jr, 1990). The model in this paper also has opportunity-spotting as a central feature of entrepreneurship, but instead of opportunity-spotting being the domain of a select few, there is an entire class of would-be entrepreneurs capable of spotting ideas—namely engineers. The basic setup of the model is most similar to Kihlstrom and Laffont (1979).

[5] Interestingly, they do not find that entrepreneurs have particularly low risk aversion, but rather use some combination of firm size, capital structure, and the possibility of default to manage downside risk.

[6] Though it is unclear whether most would-be entrepreneurs are actually liquidity constrained (Hurst and Lusardi, 2004)—they might just realize that at the terms offered by banks, most entrepreneurial ventures would be ill-advised.

Most occupational models of entrepreneurship posit some difference in individual abilities or characteristics that determines selection into entrepreneurship or employment. The individual differences that lead to entry into entrepreneurship depend on the paper: in Lucas (1978), it is managerial skill[7]; in Hayward et al. (2006), hubris; and in Lazear (2004), "balanced" skills. As already noted, in Kihlstrom and Laffont (1979), it is risk aversion, while in Holmes and Schmitz Jr (1990), it is opportunity spotting.[8] In these papers, occupational choice is still driven by financial returns, but these returns are mediated by individual characteristics, ala Roy (1951). The model in this paper also an occupational model, and it keeps the financial focus of previous work as explaining entry. However, all non-financial or behavioral factors that might explain the choice to enter entrepreneurship are set aside.

Although non-financial factors presumably do matter in practice, for innovative entrepreneurship, they matter less. For example, risk-aversion is at the heart of Kihlstrom and Laffont (1979), but for technology entrepreneurship, VC removes the individual downside risk, leaving only the opportunity cost. In Lucas (1978), managerial ability determines selection, but for technology entrepreneurship, founders must be first technically excellent, and if the startup overcomes initial technical challenges, it has the option to rely on more professional management as the startup "scales."

Extant occupation-focused models of entrepreneurship pay relatively little attention to idea selection, perhaps because for conventional entrepreneurship, the business idea is not critical.[9] In contrast, technology entrepreneurs are typically creating entirely new products, making idea selection central. There is a large, influential literature that considers the fundamental

[7]Gromb and Scharfstein (2002) has entrepreneur/managers that differ in their abilities, but the focus of the model is on the equilibrium levels of entrepreneurship and the effects this has on the relative returns to internal versus external innovation.

[8]Some work has emphasized personality attributes. For example, Levine and Rubinstein (2013) identify high intelligence but a propensity for illicit behavior; Biasi et al. (2014) document a relationship between higher risk of bipolar illness and being self-employed in an incorporated business.

[9]As Lazear (2004) writes, the "innovation [in conventional entrepreneurship] may be as seemingly minor as recognizing that a particular strip-mall would be a good location for a dry cleaner."

role of ideas in economic growth, some of which touches on entrepreneurship. This literature focuses on how economic forces (such as the possibility of business stealing by an innovating competitor) affect the incentives for costly innovation (Romer, 1990; Aghion and Howitt, 1992). Klette and Kortum (2004) introduce a Schumpeterian model of product development and improvement. More recent work in this vein includes Akcigit and Kerr (2010) and Acemoglu et al. (2013).[10]

There are some similarities between the model in this paper and the growth-focused general equilibrium models of innovation. For example, in Klette and Kortum (2004), firms also enter winner-take-all product markets tied to specific ideas, but firms arrive on their product lines by chance, and only face competition when some competitor "lands" on one of their product lines, also by chance. This assumption captures an important economic insight about the transitory nature of advantages gained through innovation, but this assumption necessarily abstracts away from how would-be entrepreneurs choose which opportunities to pursue initially, and how these decisions are shaped by the forces of competition. How the forces of competition shape what business ideas get pursued is a key focus of this paper.

One justification for the model's treatment of ideas is the nature of technology entrepreneurship makes idea competition commonplace. Technology entrepreneurs are almost always trying to exploit some *recent* technological or scientific advance. The startup's product depends critically on the advance—if the dependency were not critical, the product would not be newly possible, and it is the "newly possible" characteristic that creates the opportunity for the technology entrepreneur.[11] To the extent that there is a first-mover advantage, only *very* recent advances are worth pursuing, further concentrating technology entrepreneurs on a relatively

[10]Using a dataset on patenting behavior by firms, Akcigit and Kerr (2010) show that externally focused R&D (i.e., developing new product lines) does not scale with the firm size the same way as internal R&D (improving existing product lines). This is some justification for focusing on startups as the creators of entirely new product markets.

[11]This argument is similar to the argument for why co-discovery seems to be exceedingly common in the sciences (Merton, 1957; Lemley, 2011). The notion that new ideas come as a flow seems to have currency in sociology, e.g., Podolny et al. (1996).

small number of ideas.

3 Model

Although The model is static, it is useful to lay out the phases of entrepreneurship as if they were sequential. The phases of entrepreneurship are:

1. Some engineers select into entrepreneurship, picking a business idea to pursue; multiple entrepreneurs may pick the same idea, putting them in idea competition with each other.

2. Entrepreneurs sell equity in their startups to VCs in exchange for seed funding.

3. The world learns which startup "wins" the product market competition for each pursued idea. Some ideas are revealed to be non-viable, and no startup pursuing that idea wins.

4. Winning startups hire engineers to scale their companies, hiring from the pool of engineers that did not select into entrepreneurship.

5. Winning startups realize profits, which are split among entrepreneurs and their investors according to the ownership of equity.

In the model, many of the decisions made by the economic actors depend on the equilibrium outcome in other phases. For example, the idea quality threshold VCs use when deciding whether to fund a startup depends on the expected profits, which in turn depend on the equilibrium number of other startups funded, which depends on the idea quality threshold. As such, to illustrate the model, I will move back and forth between the different phases before deriving the model equilibrium.

3.1 Entrepreneurs select business ideas

A startup is founded by a single engineer, implementing a single idea. An idea has a probability of product market success of q distributed on $[0,1]$ with pdf $f(\cdot)$. These ideas and their respective product market success probabilities are common knowledge, and any entrepreneur is free to use any idea for his or her startup. There is a mass of $\kappa > 0$ startup ideas in total.

Suppose an idea with success probability q attracts n entrepreneurs. Only one startup of the n pursuing that idea will be successful in the product market, if that idea itself is viable. All the other $n-1$ startups are a total loss. This gives each startup pursuing that idea a product market success probability of q/n. It is essential to appreciate how idea competition works: startups are buying an idea-specific lottery ticket, and only one of the competing startups wins—and then only if the idea itself is viable.

This winner-take-all form of competition is akin to the product life cycle characterization of new goods found in Gort and Klepper (1982) and Jovanovic and MacDonald (1994), albeit with an extremely compressed "cycle." For innovative entrepreneurship, the speed with which startups learn whether they will be successful is perhaps unsurprising given the high opportunity cost of such entrepreneurs—in short, they would not pursue a startup if they could not learn their outcome relatively quickly (Arora and Nandkumar, 2011).

In the model, all ideas that are successful lead to product markets with identical characteristics.[12] Because all product markets are the same, entrepreneurs distribute themselves over ideas until the probability of individual startup success is equal for all ideas that are pursued. Let q_0 be this equalized success probability. An idea of quality q gets q/q_0 entrepreneur entrants.[13] This probability q_0 is also the quality of the "worst" idea still pursued by an entrepreneur.

[12]Note that in the model, there is no "market for ideas," and so in the Gans and Stern (2003) framework, would-be technology entrepreneurs have to choose product market competition as their commercialization strategy (rather than licensing). See Gans et al. (2002) for the implications of an environment that leads to this kind of "product market or bust" form of technical entrepreneurship.

[13]This formulation obviously has non-integer numbers of entrepreneurs per idea. This does not matter for the model predictions, but one could think of engineers allocating partial effort over a number of ideas or threatening to enter on some idea with some probability.

For a given q_0, the total number of funded entrepreneurs (and hence startups) is

$$E = \kappa \int_{q_0}^{1} \frac{q}{q_0} f(q)\,\mathrm{d}q. \tag{1}$$

The value of q_0 is determined in equilibrium, but for now, assume it is given.

The idea quality q is intended to capture both the product market viability and the systematic technical risk associated with some idea. Product market viability risk is the risk of building something no one wants to buy, while technical risk is the risk of trying to build something that no one can build (at least not yet). Entrepreneurs surely try to learn about viability and technical risk ex ante and avoid "bad" ideas, but as Kerr et al. (2014) write "for entrepreneurs, it can be virtually impossible to know whether a particular technology or product or business model will be a success until one has actually invested in it."

The relative importance of different kinds of risk presumably depends on the sector. "Hard" technology startups in fields like pharmaceuticals, bio-tech, energy, and hardware have high technical risk, but less product market viability risk. For these hard technology startups, the value of the product (if it works) is often obvious (cheaper, faster, lighter, more efficacious, etc.). In contrast, software startups—an increasingly important driver of innovation and a bright spot in the US technology sector (Arora et al., 2013)—are not especially technically risky, but there is a large risk of pursuing non-viable product markets. Paul Graham, the founder of Y Combinator (whose investment portfolio has traditionally been software focused) writes "by far, the most common mistake startups make is to solve problems no one has."[14] Consistent with these sector-specific differences, Scott et al. (2015) show that for software business ideas, experts seem to do no better than chance at "picking winners," whereas for hard technology startups, such as hardware and bio-tech, they do.

[14] An analysis of startup "post-mortems" by CBInsights, a VC industry research firm, found that the most common cause for failure was "No Market Need." Source: http://fortune.com/2014/09/25/why-startups-fail-according-to-their-founders/.

3.1.1 The elasticity of startup success probability

The key to understanding the equilibrium is understanding how the startup success probability, q_0, changes with the number of entrepreneurs. Suppose the number of entrepreneurs increases by $\mathrm{d}E > 0$. Some of these new entrepreneurs pursue ideas that were already being pursued by incumbent entrepreneurs, while others pursue "new" ideas (i.e., that have a quality $q < q_0$). Figure 1 shows the number of entrepreneurs per idea, before and after the $\mathrm{d}E$ increase. The upward sloping line $n(q) = q/q_0$ is the initial number of entrepreneurs per idea; the worst idea pursued is indicated by a vertical line at q_0. Following the $\mathrm{d}E$ increase, there are more engineers per idea (the shaded area above the sloped line) and the worst idea pursued gets shifted to $q_0 - \mathrm{d}q_0$, with a mass of entrepreneurs to the left of the original q_0.

After the $\mathrm{d}E$ shock, there is a mass of $\mathrm{d}q_0 f(q_0)\kappa$ entrepreneurs pursuing previously un-pursued ideas. Ideas that were pursued pre-shock also get more entrants: each idea of success probability q, which previously had q/q_o entrepreneurs pursuing it, now has $q/q_0^2 \,\mathrm{d}q_0$ additional entrepreneurs. The total increase in engineers pursuing already-pursued ideas is thus

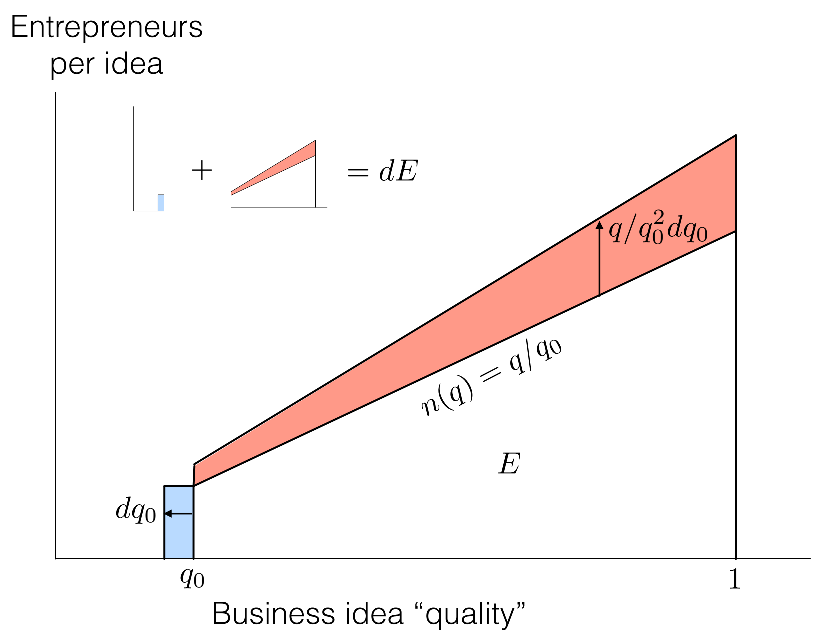
$$\kappa \int_{q_0}^{1} \left(\frac{q}{q_0^2} \,\mathrm{d}q_0 \right) f(q)\,\mathrm{d}q = E/q_0 \,\mathrm{d}q_0,$$

and so $\mathrm{d}E = \kappa f(q_0)\,\mathrm{d}q_0 + E/q_0\,\mathrm{d}q_0$. The effect of the marginal entrepreneur on q_0 (as an elasticity) is

$$\eta_E^{q_0} = -\frac{1}{1 + \kappa f(q_0) q_0 E^{-1}}. \tag{2}$$

In the expression for $\eta_E^{q_0}$, as $\kappa f(q_0) \to 0$, there are no un-pursued ideas on the "frontier," and so $\eta_E^{q_0} = -1$. This would be the case, for example, if there were some fixed number of ideas and all were already being pursued before the shock. With a completely elastic success probability, entrepreneurship is a zero-sum congestion game (Rosenthal, 1973)—adding more en-

Figure 1: The allocation of entrepreneurs over business ideas of varying quality, before and after a positive shock of dE new entrepreneurs



Notes: This figure illustrates the number of entrepreneurs per idea before and after an increase of dE new entrepreneurs. Pre-shock, each idea with quality greater than q_0 received q/q_0 entrants, where q_0 was the worst idea still funded. After entry of additional entrepreneurs, there are both (1) more entrepreneurs per idea and (2) more entrepreneurs pursuing previously un-pursued ideas, pushing down q_0.

trepreneurs does not lead to any new ideas being pursued and hence no net increase in the number of successful startups. In contrast, as $\kappa f(q_0) \to \infty$, or in elasticity terms, $\eta_E^{q_0} = 0$, more entrepreneurs do not lower startup success probabilities at all, and every new entrepreneur is pursuing a previously unexplored idea. Entrepreneurship still requires costly effort, and startups only "work" with some probability, but entrepreneurs are not in idea competition with each other, and the quality of ideas being pursued does not decline with more entry by entrepreneurs.

Another way that the startup success probability can change is through a change in κ. Suppose now that the number of entrepreneurs is unchanged, but there is a small increase in the supply of ideas, $\mathrm{d}\kappa > 0$. The elasticity of startup success probability with respect to κ has the same magnitude as the elasticity of startup success probability with respect to engineers, but opposite signs:

$$\eta_\kappa^{q_0} = -\eta_E^{q_0}. \tag{3}$$

(As the derivation is similar to the $\eta_E^{q_0}$ case, see Appendix A.1 for details.) This result will be used when deriving comparative statics related to κ. It also has implications for efficiency, as it offers a "recipe" for expanding the supply of engineers without lowering the startup success probability—namely by increasing κ at the same time the number of engineers is increased.

It is useful to consider how the distribution of perceived idea quality, $f(\cdot)$, interacts with E and κ to determine $\eta_E^{q_0}$. If would-be entrepreneurs are highly uncertain about product market viability, but know the overall average startup success rate, then $f(\cdot)$ would have a sharp peak but little mass elsewhere. This in turn could create a highly elastic or highly inelastic startup success probability, depending on where q_0 is relative to the peak of $f(\cdot)$. In contrast, if there were more variation in perceived idea quality, there would be more variation in the number of entrants per idea and a "flatter" $f(\cdot)$ and thus an $\eta_E^{q_0}$ that changed less with changes in E.

Foreshadowing the equilibrium results, Equation 2 suggests how startup costs will affect E and thus ultimately affect q_0. Industries with very high startup entry costs, such as bio-tech, pharmaceuticals, aerospace, and so on, will have fewer entrepreneurs and hence less duplicative entrepreneurship. In contrast, in an industry with low startup costs, such as software, duplicative entry would be more common because there would be so many more entrepreneurs.[15]

3.2 Demand for engineer labor and the profits of successful startups

After an entrepreneur selects an idea and obtains funding, he or she learns not only whether the idea is viable, but also whether his or her particular startup bested all other competitor startups pursuing that same idea. The ex ante probability of winning the product market for every startup is q_0, and the expected number of successful startups, collectively, is simply $q_0 E$, where E is the mass of entrepreneurs.

For each successful startup, there is some total addressable product market that offers a maximum revenue of $R > 0$. The fraction of this revenue actually obtained is $\phi(l) \in [0,1)$ where l is the mass of engineers hired as employees by that startup. I assume that for all l, $\phi'(l) > 0$ and $\phi''(l) < 0$. I also assume that $\lim_{l \to S} \phi(l) = 1$, where S is the total number of engineers. This is a technical condition that ensures an equilibrium exists, as it guarantees that if almost all engineers are working as employees, the returns to entrepreneurship will be greater than the returns to employment.

[15] As a case in point for software, Boudreau (2012), examining the software created by app developers, finds a remarkable degree of crowding, with many sellers offering more or less identical products. While many software developers creating apps might be hobbyists, this copying can and does happen on a larger scale. The German startup incubator Rocket Internet creates "dot-clones" by copying successful US Internet businesses. They have created cloned versions of eBay, Fab, Groupon, Airbnb, eHarmony, and Pinterest. Their Zappos clone, "Zalando," was worth more than $1 billion in 2012. Source: Bloomberg, "How Three Germans are Cloning the Web", February 29th, 2012. http://www.bloomberg.com/bw/articles/2012-02-29/the-germany-website-copy-machine.

Profits from a successful startup are

$$\pi = \arg\max_{l} \quad \phi(l)R - wl,$$

where w is the wage of engineer employees. Let l^* be the profit-maximizing number of employees.

Figure 2 illustrates the successful startup's choice about the optimal number of employees to hire and the resultant profits. The startup chooses l^* such that the slope at $\phi(l^*)R$ equals the wage, w, and so $\phi'(l^*) = w/R$. The vertical line extending up from l^* indicates the startup's revenue. As labor is a startup's only scaling expense, profits are simply $\phi(l^*)R - wl^*$. Because of the concavity of $\phi(\cdot)$ and the fact that employees are paid their marginal product, profits are always positive (see Appendix A.2 for the derivation). The firm-specific demand schedule for engineers is $d(w) = -\partial\pi/\partial w$.

The profit from a successful startup is common knowledge among VCs and entrepreneurs, as is whether any particular startup succeeded, and so engineer labor "scaling" costs can be paid out of anticipated revenue. Of the profits, the entrepreneur gets $e\pi$, and VCs get the rest, $(1-e)\pi$, where e is the share of equity retained by the entrepreneur. I will explain how e is determined in the next section, where I analyze the market for venture capital.

Perhaps the most consequential assumption of the model is winner-take-all in the product market. Winner-take-all/most is a common feature in technology product markets for at least two reasons. First, for some kinds of technology entrepreneurship, such as pharmaceuticals or bio-tech, patents or regulator approval ensure there is only one "winner" on each idea, with competitors racing towards winner status (Loury, 1979).[16] See Farre-Mensa et al. (2016) on the

[16] In a survey of high-technology startup entrepreneurs, Graham et al. (2009) find substantial cross-sector differences in the importance of patents, with software companies viewing patents as much less important compared to the biotechnology and medical device sectors. The usefulness of patents in the software industry is far more controversial than in other technology sectors, in that patents may be most useful in deterring entrants rather than, say, stimulating basic R&D (Cockburn and MacGarvie, 2009). There is some evidence that firms are not actually deterred by rivals having plausibly relevant patents: Mueller et al. (2013) present evidence that few firms are

importance a patent award has on subsequent funding and product market success. Second, technology startups often focus on sectors with strong network effects or scale economies. This tendency towards winner-take-all is hardly a coincidence, as VCs have little interest in backing eventual price-takers.

3.3 The market for venture capital

A would-be entrepreneur requires seed capital, c. Entrepreneurs have no other source of funds, and only a VC can provide c.[17] This investment is made once, as an arm's length investment.[18] I assume that in equilibrium, all VC firms receive the same expected return on capital, r.[19] The capital market is cleared by the share of the startup equity purchased by the VC in exchange for c. In equilibrium, the entrepreneur keeps a fraction, e, and the VC gets $1-e$ of the startup. The expected payoff from being a funded entrepreneur is thus $eq_0\pi$. The return to the capital invested in a startup equals the share of expected profits, or

$$(1+r)c = (1-e)q_0\pi. \tag{4}$$

blocked by IP, but rather pursue a variety of coping strategies, including altering projects, making acquisitions, and pursuing legal actions.

[17] One interpretation of this restriction on the source of capital is that only VCs have the expertise to know q; non-VC sources of capital would be quickly driven out of the market by adverse selection. Consistent with this view, Gompers (1995) shows that VCs concentrate their investments in high-technology firms with high information asymmetries. There is a literature exploring why there appears to be excess returns in VC. "Excess returns" in the model presented in this paper would simply mean that r is higher than what we would expect given the payoff π and probability of success, q_0. Sorensen (2007) offers a two-sided matching model of venture capital. Excess returns to are due to sorting. Jovanovic and Szentes (2013) attribute the excess returns to market power by VCs. Kurlat (2015) explores the social efficiency of financial expertise and offers a sufficient statistic for whether this expertise has a socially beneficial effect.

[18] This contrasts with Bergemann and Hege (2005), who model funding as a flow rather than a one-off investment.

[19] In a survey of the VC literature, Da Rin et al. (2011) note: "Overall we note that while different studies obtain somewhat different estimates of the net returns, there is an emerging consensus that average returns of VC funds do not exceed market returns." Relying on a dataset of firms receiving multiple term-sheet offers from VCs, Hsu (2004) finds that startups offer a substantial discount to "brand-name" VCs. It is unclear whether this should be interpreted as market power or as VCs offering a bundled service of capital and expertise that entrepreneurs "pay" for with lower valuations.

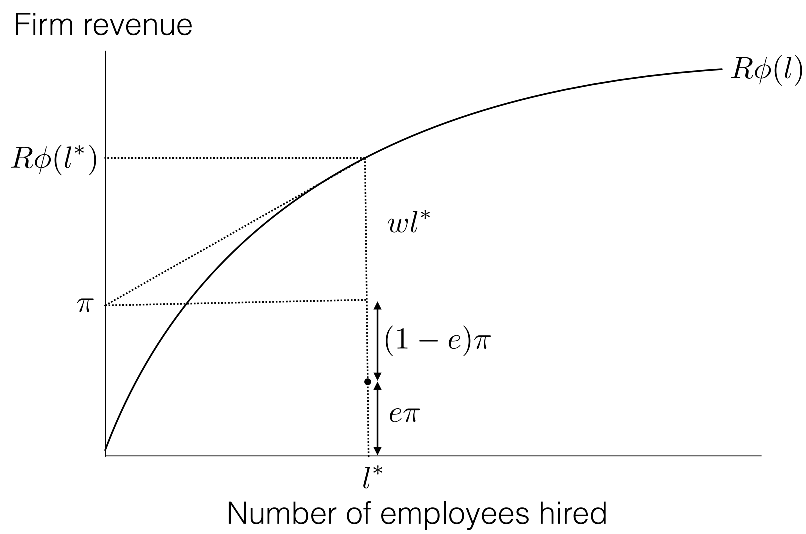
Let $C = (1+r)c$. This C is the cost of founding a startup, both direct and financial, but does not include the entrepreneur's opportunity cost. As all startups have the same success probability, q_0, and all offer the same product market if successful, VCs are indifferent over the startups that are funded in equilibrium. An unfunded startup could be thought of as a firm proposing to be the $n+1$ firm on an idea that already has $q/n = q_0$ (a rejected "imitator") or proposing a q' idea where $q' < q_0$ (a rejected "innovator").[20]

Note that VCs get an *expected* return of r. VCs get a return of $1/q_0$ on their initial investments with probability q_0 and a return of 0 with probability $1-q_0$, and so the variance in the return is $(1-q_0)/q_0$. For sectors with low startup success probabilities, the model can generate the highly skewed pattern of returns typical in the VC industry. For example, if $q_0 = 0.05$, successful VCs enjoy 20X returns on their investments, though this only happens for 1/20th of the investments made.

Given the pattern of returns for startups, VCs invariably pursue a portfolio strategy. In the model equilibrium, VCs would be just as happy with some other portfolio as their own, as they are indifferent over funded startups. Of course, VCs do presumably see advantages in their particular investments, but there is little evidence that having picked "better" ideas is even their own justification. Bernstein et al. (2016) show that potential investors are far more responsive when evaluating pitches to the characteristics of the founding team than any other source of information. Some VCs have gone so far as to claim that "ideas don't matter." Coming from VCs, this is perhaps self-serving, but the insight of the "ideas don't matter" perspective is that: (1) there are lots of ideas "in the air," and (2) any obviously good idea will attract lots of entrants, so execution is paramount.

[20] Hellman and Puri (2000) construct a dataset of Silicon Valley startups and find that firms they label as "innovators" are more likely to receive funding than "imitator" firms.

Figure 2: Successful startup's demand for engineer-employee labor



Notes: This figure illustrates the successful startup's hiring problem. The curve labeled $R\phi(l)$ is the revenue obtained from hiring l engineer employees. The firm chooses l^* such that $R\phi(l^*)$ is tangent to w, the market wage of engineer employees. The resultant profits are split between the entrepreneur, who gets a share e, and his or her VC investor, who gets $1-e$.

3.4 The returns to entrepreneurship and employment

With the capital and product markets modeled, I can now turn to the career decision of engineers, who must choose between employment and entrepreneurship. I assume that engineers have to pick an occupation ex ante; one could also think of engineer employees as those that could not obtain funding in the VC market and thus have to pursue employment.

For an engineer to be indifferent between occupations, the expected payoff from entrepreneurship must equal the wage as an employee, and so

$$\begin{aligned} w &= eq_0\pi \\ &= q_0\pi - C. \end{aligned} \tag{5}$$

The ratio of entrepreneur equity to VC equity is the ratio of the entrepreneur's opportunity cost to the VCs' opportunity cost, or $e/(1-e) = w/C$. Note also that the retained equity of an entrepreneur is $e = w/(w+C)$. As startup costs go to zero, entrepreneurs retain full ownership, as VCs become superfluous.

Note that in the model, the realized returns for the individual entrepreneur is a random variable—with probability q_0 he or she gets $e\pi$ and with probability $1-q_0$, he or she gets 0. He or she also faces an opportunity cost of w. However, unlike the VC, he or she has no capital at risk. For many kinds of technology entrepreneurship, C is far from trivial, and this reduction in downside risk is important to understanding why technology entrepreneurship exists despite the phenomenal risks—Hall and Woodward (2010) find that the modal return to venture-backed entrepreneurship is \$0 but that the expected value for the individual entrepreneur would exceed all but the highest salaries.

The elimination of downside risk for individual entrepreneurs via VC is perhaps the defining institutional feature of technology entrepreneurship. The availability of unsecured funds for technology entrepreneurship is the exception that proves the rule, with the rule being that entrepreneurship is personally risky. The special features of technology entrepreneurship account for the difference, namely (1) the large potential upside to technology business ideas and (2) the special human capital needed for technology entrepreneurship. Few individuals with the requisite human capital for technology entrepreneurship would also have the necessary financial capital. VC is the economic solution to this problem. In terms of the model, the prevalence of VCs and the unsecuritized funding they offer to would-be technology entrepreneurs is the reason I set aside risk-aversion as a factor in explaining occupational choice, but as discussed earlier, this would be an untenable assumption for conventional entrepreneurship.

In extant occupational models of entrepreneurship, individual differences lead to sorting, and all inframarginal individuals get a surplus from their preferred occupation. As such, we would expect only the marginal individual to move between occupations when the returns change slightly. Technology entrepreneurs are disproportionately drawn from employees of successful ventures of a slightly older vintage (Gompers et al., 2005; Klepper and Sleeper, 2005; Chatterji, 2009). Perhaps these are the "marginal" engineers not strongly attached to employment or entrepreneurship. However, as I show in (removed for the purposes of review), movement back and forth between entrepreneurship and employment is widespread among elite engineers. This is hard to reconcile with individuals having specialized skills for either employment or entrepreneurship which drive occupational sorting. In short, no one looks inframarginal, as we would expect if individuals have strong comparative advantages in different occupations.

A reconciliation of the frequent back-and-forth, and the point about inframarginality made above, is that there is little first order difference between elite engineer employees and founders. To the extent that some taste or ability differences drive selection, they are secondary in importance to understanding selection into technology entrepreneurship. Job hopping is common in technology hubs like Silicon Valley (Fallick et al., 2006), and while much of this movement is between companies, occupation-type jumping seems common as well. This movement creates the possibility "occupational tâtonnement" that would tend to equalize returns between employment and entrepreneurship, which motivates Equation 5.

3.5 Market clearing

There is a mass of S engineers, all of which participate as either employees or entrepreneurs. Those who participate as employees supply labor inelastically on both the intensive and extensive margins.[21] Each startup claims one engineer, the entrepreneur. Each successful startup also claims l^* additional engineers as employees. The total number of engineer employees is $q_0 E l^*$. As $S = E + L$, and so the fraction of engineers pursuing entrepreneurship in equilibrium is

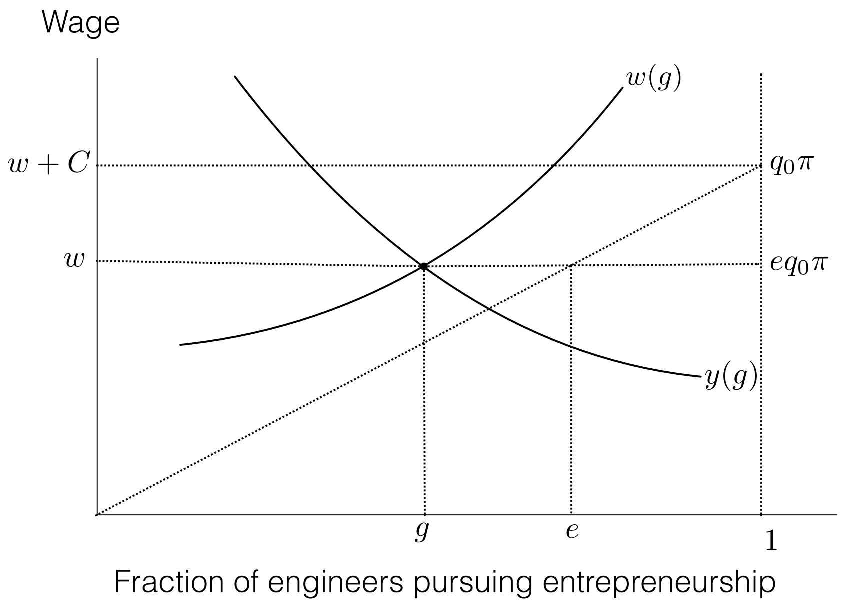
$$g = \frac{1}{1 + q_0 l^*}. \tag{6}$$

The fraction of engineers pursuing entrepreneurship is an equilibrium outcome. However, it useful to think about g as if it were exogenously set in order to understand the model equilibrium. Figure 3 shows the returns to employment, $w(g)$, and returns to entrepreneurship, $y(g)$, as functions of g. As I will show formally later, the wage is increasing in g, and the return to entrepreneurship is decreasing in g, i.e., $w'(g) > 0$ and $y'(g) < 0$.

The economic argument for decreasing returns in each occupation is straightforward. If entrepreneurship were rationed and only some fraction $g_L < g$ of engineers were allowed to participate, there would be more supply in the labor market, lowering wages and raising profits for entrepreneurs; those lucky entrepreneurs would also benefit from less idea competition from

[21] Modeling the number of engineers as a continuum avoids the complexity of a finite number of entrepreneurs, which would make the realized number of firms a random variable. This is not a consequential assumption, in that even if the number of successful firms was a binomial random variable, the resultant engineer wage—which is what both would-be entrepreneurs and employees care about—would converge to a constant as S grew large. To see why, suppose that N_A is the realized number of startups and $N_A \sim \text{Binom}(N, q_0)$. The resulting wage would have to satisfy $d(w)N_A \equiv (1-g)S$, and so $w = d^{-1}(X)$, where $X = \frac{(1-g)S}{N_A}$. The variance of X is $V(X) = \frac{(1-g)^2(1-q_0)}{g^2 q_0^2 S}$, thus $\lim_{S\to\infty} V(X) = 0$ (recall that q_0 and g are fixed).

Figure 3: Equilibrium allocation of engineers to employment and entrepreneurship



Notes: This figure shows the returns to employment, $w(g)$, and entrepreneurship, $y(g)$, as functions of g, the fraction of engineers that pursue entrepreneurship. At the equilibrium wage, labeled w, returns are equal. The expected profits from entrepreneurship, $q_0\pi$, are equal to $w + C$, so the dashed line from the origin to the upper-right corner labeled πq_0 has a slope of πq_0 and thus intersects the horizontal line (at w) at the point e on the horizontal axis.

other startups, and so would have a higher success probability. Similarly, if employment were rationed, with $g_H > g$, the entrepreneurs would fare worse than their employee counterparts because of higher labor costs and increased competition from other entrepreneurs. Intuitively, it is the decreasing returns to an occupation as more engineers enter that occupation that leads to an equilibrium.

Proposition 1 asserts that the model has an equilibrium and that it is unique, so long as C/R is sufficiently small. The restriction on C/R ensures that entrepreneurship is profitable when almost no other engineers pursue it.

Proposition 1. *With a sufficiently small ratio of startup costs to potential revenue, C/R, a model equilibrium exists and is unique.*

Proof. See Appendix A.4. □

One additional feature of the model illustrated by Figure 3 is the equity retained by entrepreneurs, e, in equilibrium. Drawing a horizontal line at $w+C$ indicates the expected profits. As the horizontal axis has unit length, the slope from the origin to $(1, q_0\pi)$ is $1/(q_0\pi)$. This sloped line intersects the w horizontal line at the point $(e, 0)$, as $e\pi q_0 = w$. The construction of e shows why, graphically, a reduction in startup costs increases ownership by entrepreneurs—a lower $w+C$ pushes e to the right.

3.6 The effects of the marginal entrepreneur

If a shock to the entrepreneurial system changes the relative returns to the two occupations, engineers flow away from the disadvantaged occupation until returns are equalized. For example, if some shock makes entrepreneurship costlier but leaves the labor market unaffected, engineers will flow from entrepreneurship to employment.[22] The magnitude of the flow depends on the size of the "gap" created by the shock at the original equilibrium. The bigger the original gap, the more movement needed. It also matters how effective flows are in shrinking the gap; if the financial returns are highly elastic, relatively small flows are needed, and vice versa when returns are inelastic.

To obtain the comparative statics of the model, there are two analytical steps: (1) determine the magnitude and direction of the shock at the original equilibrium, and (2) determine the direction and magnitude of the flow of engineers needed to restore an equilibrium.

For (1), different kinds of shocks have different effects. For example, before engineers adjust occupations, startup costs affect entrepreneurship while leaving the labor market unchanged, but changes in the extent of the product market affect both employment and entrepreneurship directly, at the same time. As such, these different shocks must be worked out one by one for

[22] Shocks to the total supply of engineers, S, is qualitatively different, but can be analyzed by assuming that new entrants initially split in the same proportions as pre-shock engineers.

that kind of shock. For (2), the adjustment that comes from engineers flowing between employment and entrepreneurship is the same regardless of the shock, and so this can be worked out just once, which I will do now.

Suppose there is a shock to the number of employees, $\mathrm{d}L < 0$, and a positive shock to the number of entrepreneurs, $\mathrm{d}E > 0$. In the labor market, supply goes down. Demand goes up, if at least some of the $\mathrm{d}E$ new entrepreneurs create successful, labor-demanding startups. Figure 4 illustrates the shock. Because workers supply labor inelastically, the supply curve is vertical and is moved to the left by the shock, going from L_0 to L_1, with $\mathrm{d}L = L_1 - L_0$. With $\mathrm{d}E$ more entrepreneurs, the demand curve shifts out by $(\partial D/\partial E)\,\mathrm{d}E$, going from D_0 to D_1. Wages rise, going from w_0 to w_1. The magnitude of the increase in wages depends on the slope of the demand curve, $\partial D/\partial w$. The net effect of the shock on the wage is

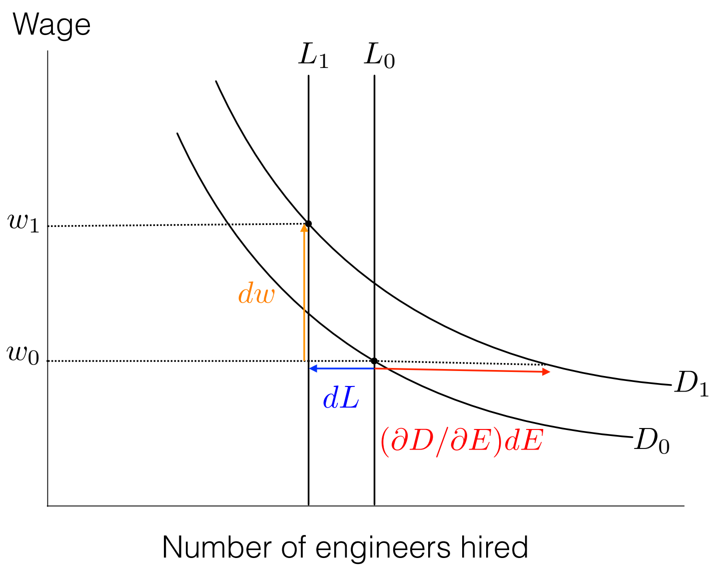
$$\mathrm{d}w = \frac{(\partial D/\partial E)\,\mathrm{d}E - \mathrm{d}L}{|\partial D/\partial w|}. \tag{7}$$

The magnitude of $\partial D/\partial E$ depends on the probability that a marginal entrepreneur pursues a previously un-pursued idea. Entrepreneurs that pursue already-pursued ideas only crowd out incumbent entrepreneurs, leading to no net increase in successful startups, and hence no increase in labor demand. As the labor demand curve is $D = q_0 E d(w)$, the number of entrepreneurs enters linearly. However, E also affects q_0, which is lowered when more entrepreneurs enter the system (recall Equation 2). As such, the elasticity of labor demand with respect to the number of entrepreneurs is

$$\eta_E^D = 1 - |\eta_E^{q_0}|. \tag{8}$$

(See Appendix A.3 for the derivation.) If $\eta_E^{q_0} = -1$, increasing the number of entrepreneurs would have no effect on labor demand, as the number of successful startups would not increase. In contrast, if $\eta_E^{q_0} = 0$, every marginal entrepreneur is pursuing a previously un-pursued idea. In

Figure 4: Labor market for engineers following a small decrease in the number of employees and a small increase in the number of entrepreneurs



Notes: This figure illustrates the labor market for employees following a small decrease in the supply of engineers, $\mathrm{d}L < 0$, and a small increase in entrepreneurs, $\mathrm{d}E > 0$. The supply curve shifts from L_0 to L_1 and the demand curve shifts from D_0 and D_1. The magnitude of the shift in the demand curve depends on the number of new firms the marginal entrepreneurs create, $\partial D/\partial E$. This in turn depends on the probability that a marginal engineer pursued an idea that was previously pursued by an incumbent.

this case, the "full" demand effect is felt in the labor market when the number of entrepreneurs increases, and so $\eta_E^D = 1$.

Although I have treated $\mathrm{d}E$ and $\mathrm{d}L$ as independent shocks, if the total supply of engineers is unchanged, these shocks are equal in magnitude but opposite in sign. In this case, $\mathrm{d}E = -\mathrm{d}L$, and as $L = (1-g)S$ and $E = gS$, $\mathrm{d}L = -S\mathrm{d}g$ and $\mathrm{d}E = S\mathrm{d}g$. With these definitions of the shocks, the effects of a flow into entrepreneurship can be characterized in terms of changes in g. Using Equation 7, the elasticity of the wage with respect to the fraction of engineers pursuing entrepreneurship is

$$\eta_g^w = -\frac{1}{\eta_w^D}\left(\eta_E^{q_0} + \frac{1}{1-g}\right). \tag{9}$$

Note that as $\eta_w^D < 0$, $g < 1$ and $|\eta^{q_0}| \leq 1$, if the number of entrepreneurs is increased but all else stays the same, wages rise, or $\eta_g^w > 0$.

From the perspective of the entrepreneur, the $(\mathrm{d}L, \mathrm{d}E)$ shock affects them in three ways: (1) the profits of a successful startup decrease because of the higher labor costs, (2) the startup success probability falls because of the additional entrepreneurs, and (3) the share of retained equity falls because VCs require a larger share of equity in exchange for c, given that expected profits have fallen. The net effect is $\eta_g^y = \eta_E^e + \eta_E^{q_0} + \eta_E^\pi$. Note that $\eta_E^i = \eta_g^i$, for $i \in \{e, q_0, \pi,\}$, as $g = E/S$. As $w = \pi q_0 - C$, the elasticity of entrepreneurship returns can also be written as $\eta_g^y = \frac{1}{e}\left(\eta_E^{q_0} + \eta_E^\pi\right)$.

Profits only change because of the change in labor costs, and so $\eta_g^\pi = -eq_0 l^* \eta_g^w$. The elasticity of the returns to entrepreneurship with respect to g can thus be written as

$$\eta_g^y = \frac{1}{e}\eta_E^{q_0} - \frac{1-g}{g}\eta_g^w. \tag{10}$$

As $\eta_E^{q_0} < 0$ and $\eta_g^w > 0$, the return to entrepreneurship is decreasing in the number of entrepreneurs,

$\eta_g^y < 0$. The difference in the elasticities will be used frequently, and can be written as

$$\eta_g^w - \eta_g^y = \frac{1}{g}\eta_g^w - \frac{1}{e}\eta_E^{q_0}. \tag{11}$$

As $\eta_g^y < 0$ and $\eta_g^w > 0$, the difference is positive. Note that the entrepreneurship returns curve from Figure 3 is always steeper than the employment returns curve, as $|\eta_g^y| > |\eta_g^w|$.

3.7 The flow of engineers to re-establish an equilibrium

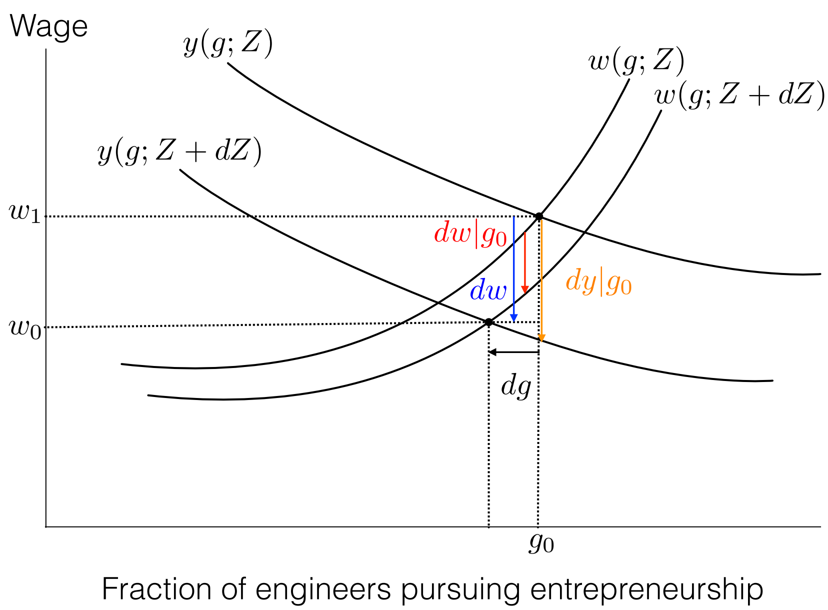
The previous section showed how a flow of entrepreneurs altered the return to employment and entrepreneurship. Now I consider a shock that would make a flow necessary to re-establish an equilibrium.

Consider a shock, $\mathrm{d}Z$, that affects the returns to employment and entrepreneurship, before any engineer has changed occupations. Figure 5 illustrates the effects of the $\mathrm{d}Z$ shock on the equilibrium. Let the pre-adjustment change in the wage be $\mathrm{d}w|g_0$ and the change in returns to entrepreneurship be $\mathrm{d}y|g_0$. These are the vertical shifts in the employment and entrepreneurship return curves. The wage curve shifts from $w(g;Z)$ to $w(g;Z+\mathrm{d}Z)$, with $\mathrm{d}w|g_0 = w(g_0;Z+\mathrm{d}Z) - w(g_0;Z)$. The entrepreneurship returns curve shifts from $y(g;Z)$ to $y(g;Z+\mathrm{d}Z)$, with $\mathrm{d}y|g_0 = y(g_0;Z+\mathrm{d}Z) - y(g_0;Z)$. Let $\eta_Z^{w|g_0}$ and $\eta_Z^{y|g_0}$ be the point elasticities of w and y with respect to Z, at the original, pre-shock entrepreneurial fraction, g_0.

The new equilibrium arises when $\mathrm{d}g$ engineers shift, which changes the equilibrium wage by $\mathrm{d}w = w_1 - w_0$. As drawn, the shock requires a flow of size $\mathrm{d}g$ from entrepreneurship to employment to reestablish an equilibrium. There are two ways of expressing $\mathrm{d}w$:

$$\begin{aligned}
\mathrm{d}w &= \mathrm{d}w|g_0 + \frac{\partial w(g)}{\partial g}\frac{\partial g}{\partial Z}\mathrm{d}Z \\
&= \mathrm{d}y|g_0 + \frac{\partial y(g)}{\partial g}\frac{\partial g}{\partial Z}\mathrm{d}Z.
\end{aligned}$$

Figure 5: Occupation choice following a shock to the returns to employment and entrepreneurship



Notes: This figure illustrates the effects of a shock, dZ, on the equilibrium occupational choice of engineers. Pre-shock, the fraction of engineers pursuing entrepreneurship is g_0. With the dZ shock, the wage curve shifts from $w(g;Z)$ to $w(g;Z+\mathrm{d}Z)$. The change in wages at the original allocation, g_0, is $\mathrm{d}w|g_0 = w(g_0;Z+\mathrm{d}Z) - w(g_0;Z)$. A similar change occurs for entrepreneurship, with $\mathrm{d}y|g_0 = y(g_0;Z+\mathrm{d}Z) - y(g_0;Z)$. With the particular shock that is shown, when engineers adjust occupations, dg flow into employment and the equilibrium change in the wage is dw.

Using these two ways of expressing $\mathrm{d}w$, the elasticity of the entrepreneurial fraction with respect to the shock to Z is

$$\eta_Z^g = \frac{\eta_Z^{w|g_0} - \eta_Z^{y|g_0}}{\eta_g^w - \eta_g^y}. \tag{12}$$

The elasticity of the wage with respect to Z is

$$\eta_Z^w = \frac{\eta_g^y \eta_Z^{w|g_0} - \eta_g^w \eta_Z^{y|g_0}}{\eta_g^w - \eta_g^y}. \tag{13}$$

These expressions simplify the comparative statics analysis: I can first determine what a shock does to the returns of employees and entrepreneurs before any engineer can change occupations, and then the pre-adjustment changes in returns can then be plugged into Equations 12 and 13 to get the net effects of the shock on g and w.

3.8 The effects of changes in startup costs

Consider an increase in startup costs, $\mathrm{d}C > 0$. This could be due to, say, a more difficult market for startup funding. The most common source of changes is likely technological innovation that would tend to lower startup costs.

When startup costs increase, before engineers can adjust occupations, the entrepreneurship return curve, $y(g)$, is shifted down by $\mathrm{d}C$. Employees are unaffected, and so the $w(g)$ curve does not move. In terms of the notation developed in Section 3.7, $\mathrm{d}w|g_0 = 0$ and $\mathrm{d}y|g_0 = -\mathrm{d}C$, and so the point elasticity of the entrepreneurial fraction with respect to startup costs is

$$\eta_C^g = -\frac{C/w}{\eta_g^w - \eta_g^y}. \tag{14}$$

As $\eta_g^w - \eta_g^y > 0$, a rise in startup costs causes engineers to flow from entrepreneurship into employment, i.e., $\eta_C^g < 0$. When $\eta_g^w - \eta_g^y$ is large, a relatively a small flow of entrepreneurs is sufficient to re-establish an equilibrium, and vice versa when $\eta_g^w - \eta_g^y$ is small.

The equilibrium change in wages with respect to a change in C is

$$\frac{\partial w}{\partial C} = g\left(-1 + \frac{1}{1-e}\eta_E^{q_0}\eta_C^g\right), \tag{15}$$

which we can re-write as

$$\frac{\partial w}{\partial C} = g\left(-1 + \frac{1}{e}\frac{|\eta_E^{q_0}|}{\eta_g^w - \eta_g^y}\right) \tag{16}$$

When g is relatively high, meaning lots of engineers are entrepreneurs, changes in C have a large effect on w. When there are relatively few entrepreneurs but many employees (low g), higher startup costs have a smaller effect on wages since there is less total increase in startup costs to be borne by engineers as a group.

The term inside the parentheses has an economic interpretation, in that it captures how large a flow of engineers must leave entrepreneurship to re-establish an equilibrium, with larger flows leading to greater reductions in wages.

Suppose that the startup success probability was completely inelastic, $|\eta_E^{q_0}| = 0$. The increase in startup costs drives engineers from entrepreneurship, but because the startup success probability does not change, there is no compensating increase in success probability that would occur if $|\eta_E^{q_0}| > 0$. As such, a larger flow out of entrepreneurship is needed to re-establish the equilibrium, which means that employees see a larger fall off in wages. With a highly elastic success probability, a smaller number of exiting engineers is needed to establish a new equilibrium, and so there is less downward wage pressure and so less pass through of startup costs.

Proposition 2 summarizes the results already discussed and contains predictions about the other outcomes of interest following a change in startup costs.

Proposition 2. *An increase in startup costs: (1) lowers the wages of engineers, (2) lowers the retained equity of entrepreneurs, (3) raises the startup probability of success, (4) raises expected profits, (5) raises realized profits, and (6) reduces the fraction of engineers pursuing entrepreneurship.*

Proof. See Appendix A.6. □

3.9 Supply of engineers

Consider an increase in the supply of engineers, $\mathrm{d}S > 0$. Assume that these $\mathrm{d}S$ engineers initially split into employment and entrepreneurship in the same proportion as existing engineers: $(1-g)\,\mathrm{d}S$ join the labor market, while $g\,\mathrm{d}S$ become entrepreneurs.

The effects of this shock in the labor market are shown in Figure 6. The supply curve shifts from L_0 to L_1, while the demand curve shifts from D_0 to D_1. The elasticity of the engineer wage with respect to the supply shock at the original entrepreneurial fraction is

$$\eta_S^{w|g_0} = -\frac{\eta_E^{q_0}}{\eta_w^D}. \tag{17}$$

Despite there being more entrepreneurs, which increases demand, wages fall with a supply shock, as $\eta_S^w|_{\mathrm{d}g=0} < 0$.

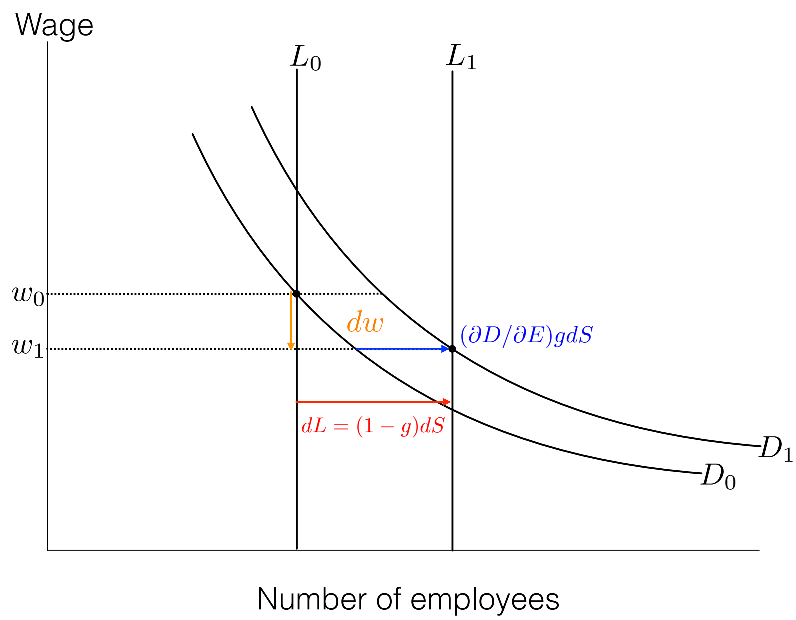
The elasticity of entrepreneur returns, before occupational adjustment, is

$$\eta_S^{y|g_0} = -q_0 l^* \eta_S^{w|g_0} + \frac{1}{e}\eta_E^{q_0}. \tag{18}$$

After the initial shock, engineers flow to the relatively more advantaged occupation. The elasticity of the entrepreneurial fraction with respect to S is

$$\eta_S^g = \left(\frac{1}{g\eta_w^D} + \frac{1}{e}\right)\frac{\eta_E^{q_0}}{\eta_g^w - \eta_g^y}.$$

Figure 6: Effects of a positive engineer supply shock on the labor market when new entrants split into occupations in same proportions as incumbent engineers



Notes: This figure shows the labor market effects of an increase in the total number of engineers, when these new engineers pursue employment and entrepreneurship in the same proportion as incumbents. This shock shifts the supply curve out by $\mathrm{d}L = (1-g)\,\mathrm{d}S$. It also shifts out the demand curve, with the size of the movement depending on how many of the new entrant engineers create additional successful firms.

The sign of η_S^g tells us whether new entrants are biased towards entrepreneurship or employment. As $\eta_E^{q_0} < 0$, the shock is biased towards entrepreneurship when $|\eta_w^D| < e/g$. If labor demand is highly elastic, when S goes up, these new engineers can be "absorbed" in the labor market with little reduction in wages, and so the supply shock tends to be employment-biased. The reverse is true if labor demand is highly inelastic, as wages quickly fall with more employees.

The elasticity of engineer wages to a supply shock is

$$\eta_S^w = \left(\frac{g}{e}\right)\eta_E^{q_0}\left(1+\eta_S^g\right). \tag{19}$$

As $\eta_E^{q_0} < 0$, $\eta_S^w < 0$ if $1+\eta_S^g > 0$, which is the case: even if all new engineers entered employment, the elasticity is $\lim_{dS\to 0} \frac{\frac{E}{S+dS}-\frac{E}{S}}{(dS)/S} = -g$ and $g < 1$. Although a supply shock always lowers engineer wages, the amount depends in part on the elasticity of startup success probability with respect to entrepreneurs. When $\eta_E^{q_0} = 0$, engineer wages do not fall at all with more supply, as $\eta_S^w = 0$. The reason is that in the absence of idea competition, the returns to entrepreneurship do not decrease with more entrepreneurs, and so the entering engineers do as well as incumbent engineers did, pre-shock. When the startup success probability is inelastic, adding more engineer labor supply is a nearly free lunch; new entrants do not lower the wages of incumbent engineers or hurt incumbent entrepreneurs, but they do increase the number of successful startups.

Proposition 3 summarizes the other comparative static results for a positive supply shock of engineers.

Proposition 3. *An increase in the supply of engineers: (1) lowers the wages of engineers, (2) lowers the retained equity of entrepreneurs, (3) lowers expected profits, (4) raises realized profits, (5) lowers the startup probability of success, and (6) has an ambiguous effect on entrepreneurship.*

Proof. See Appendix A.8. □

3.10 Supply of ideas

Consider a positive shock to the supply of ideas, $d\kappa > 0$, before engineers can adjust occupations. This increases the "space" of ideas for would-be entrepreneurs, which reduces crowding and thus raises the startup success probability. This would seemingly increase the appeal of entrepreneurship, but with reduced crowding on ideas, there are more successful startups and hence increased demand for labor. As such, the relative increase in returns is what matters for predicting the flow of engineers post-shock, as was the case with the supply shock.

The effect of more ideas on wages at the original entrepreneurial fraction is

$$\begin{aligned}\eta_\kappa^{w|g_0} &= -\eta_\kappa^{q_0|g_0}/\eta_w^D \\ &= \eta_E^{q_0}/\eta_w^D.\end{aligned}$$

The effect on the returns to entrepreneurship is

$$\begin{aligned}\eta_\kappa^{y|g_0} &= -q_0 l^* \eta_\kappa^{w|g_0} + \frac{1}{e}\eta_\kappa^{q_0|g_0} \\ &= -q_0 l^* \eta_\kappa^{w|g_0} - \frac{1}{e}\eta_E^{q_0}.\end{aligned}$$

After engineers adjust occupations, the net effect on the entrepreneurial fraction is

$$\eta_\kappa^g = -\left(\frac{1}{g\eta_w^D} + \frac{1}{e}\right)\frac{\eta_E^{q_0}}{\eta_g^w - \eta_g^y}.$$

Whether entrepreneurship increases or decreases depends on whether $|\eta_w^D| < e/g$, just as in the case of a supply shock. The difference is that with the idea shock, both wage and entrepreneurship returns curves are shifting up. If labor demand is highly elastic, when κ goes up, the additional startups that result do not raise wages very much, and so the shock is biased towards entrepreneurship. The reverse is true if labor demand is highly inelastic, as the additional startups have a large positive effect on wages. The net effect on wages from a positive ideas shock, after entrepreneurs adjust, is

$$\eta_\kappa^w = -\left(\frac{g}{e}\right)\eta_E^{q_0}\left(1-\eta_\kappa^g\right). \tag{20}$$

As $\eta_E^{q_0} < 0$, $\eta_k^w > 0$, and so wages rise following a positive idea shock. Note that the larger $|\eta_E^{q_0}|$, the bigger the effect on wages. This simply reflects the fact that when startup success probability is highly elastic with respect to E, it is also highly elastic with respect to κ, as these two elasticities have the same magnitude: $\eta_\kappa^g = -\eta_S^g$ and $\eta_\kappa^w = -\eta_S^w$, mirroring the earlier results about the equivalence of changes in E and changes in κ on the startup success probability. When there is lots of idea competition, increasing κ is particularly welcome. Interestingly, if $\eta_E^{q_0} = 0$, then creating more ideas would not increase wages for the simple reason that the marginal engineer pursues ideas that already were previously unpursued and so adding more unpursued ideas does nothing.

For all the other shocks considered so far, the startup success probability was only affected through the channel of engineers entering or exiting entrepreneurship. However, in the case of the supply of ideas, the idea supply shock has a separate, independent effect on success probability. The net effect is

$$\frac{\partial q_0}{\partial \kappa} = \frac{\partial q_0}{\partial g}\frac{\partial g}{\partial \kappa} + \left.\frac{\partial q_0}{\partial \kappa}\right|_{g_0},$$

or in terms of elasticities, $\eta_\kappa^{q_0} = |\eta_E^{q_0}|(1-\eta_\kappa^g)$.

Although wages always rise when κ increases at the original g_0, it is possible that the returns to entrepreneurship fall before the adjustment, i.e., $\eta_\kappa^{y|g_0} < 0$. This can occur when labor costs are high, and those costs are largely passed through to entrepreneurs. For example, if e is close to 1, q_0 and l^* are large, and demand is very inelastic, an increase in demand from a higher κ could cause a rise in wages high enough to lower the returns to entrepreneurship. However, when engineers can adjust, both w and y have to rise. The reason is that in a scenario where $\eta_\kappa^{y|g_0} < 0$, entrepreneurs would flow out of entrepreneurship. This would make $\eta_\kappa^g < 0$, and from Equation 20, wages would have to increase (and hence y would be higher in equilibrium as well).

Proposition 4 summarizes the comparative static results from a positive shock to κ.

Proposition 4. *An increase in the stock of startup ideas: (1) raises the wages of engineers, (2) raises the retained equity of entrepreneurs, (3) raises expected profits, (4) lowers realized profits, (5) raises the startup probability of success, and (6) has an ambiguous effect on entrepreneurship.*

Proof. See Appendix A.9. □

From a policy perspective, focusing on the supply of ideas is attractive for several reasons. First, there is a pre-existing policy lever that can be continuously adjusted—namely the direct subsidization of R&D (Jaffe, 1989). Because of the public good nature of research, crowding out private funding is unlikely, which is a concern with, say, directly funding startups to lower C. If anything, crowding in is more likely for R&D, as an emerging literature documents a kind of R&D multiplier of sorts (Moretti et al., 2014; Azoulay et al., 2014)—government R&D seems to cause more R&D spending by nearby industrial labs.

Although subsidizing R&D is a policy lever, it is of course difficult to know if it is changing the κ of the system, which is not directly observed. Patents would seem like a reasonably proxy, but much of the knowledge generated in research labs seems to spill out; Agrawal and Henderson (2002) finds that in the case of the Mechanical and Electrical Engineering Department at MIT, researchers claimed that less than 10% of the knowledge transferred out of their labs was patented. These "leaks" are fine from a social standpoint, and un-patented work is probably easier for other startups to commercialize, though this complicates trying to calculate the ROI of this sort of spending.

From a purely descriptive standpoint, exogenous changes in κ could explain the boom-or-bust pattern that seems common in technology clusters. In short, some combination of existing technologies leads to a technological breakthrough ala Fleming (2001), rapidly expanding the space of potential business ideas which then are exhausted by innovating entrepreneurs exploring the commercial implication of the advance.

3.11 Extent of the product market

Consider an increase in the extent of the product market, $\mathrm{d}R > 0$. Before engineers can change occupations, the effect on the wages of employees is $\frac{\partial w}{\partial R}\big|g_0 = \phi'(l^*) = w/R$. As such, $\eta_R^{w|g_0} = 1$. For the successful entrepreneur, the effect on profits is $\frac{\partial \pi}{\partial R}\big|g_0 = \pi/R$, and so $\eta_R^{\pi|g_0} = 1$. However, what matters to entrepreneurs is The effect on expected profits, which depends on the effect on e and q_0. Without any occupational adjustment, q_0 stays the same. The case of equity is different: as VCs only make a return on the startup costs, their share of equity falls, as profits are larger. The entrepreneur's equity increases by $\frac{\partial e}{\partial R}\big|g_0 = (1-e)/R$. The overall effect on the entrepreneur's return from an expansion of the product market is $\eta_R^{y|g_0} = 1/e$.

With occupational adjustment, both $\eta_R^{w|g_0}$ and $\eta_R^{y|g_0}$ are positive, and so wages rise and engineers are shifted towards entrepreneurship. The elasticity of the entrepreneurial fraction with respect to the extent of the product market is

$$\eta_R^g = \frac{(1-e)/e}{\eta_g^w - \eta_g^y}.$$

As $e > 0$ and $\eta_g^w - \eta_g^y > 0$, $\eta_R^g > 0$.

An increase in the extent of the product market draws engineers into entrepreneurship. Interestingly, this elasticity is the same magnitude—but opposite sign—as that from Equation 14, which gave the expression for η_C^g, or the elasticity of the entrepreneurial fraction with respect to the startup cost (note that $(1-e)/e = C/w$). In other words, a 10% expansion in revenue available in the product market has the same effect as a 10% reduction in the costs of doing a startup.

The expression for the effect on the wages of engineers following adjustment is

$$\eta_R^w = 1 + \eta_g^w \eta_R^g. \tag{21}$$

As $\eta_g^w > 0$ and $\eta_R^g > 0$, the elasticity of wages with respect to the extent of the product is not only positive, but it also has an elasticity greater than 1. When the product market expands, employees get a percentage wage increase the same size as the product market expansion. But then as the shock draws engineers into entrepreneurship, those employees that stay also benefit from the tighter labor market, receiving even higher wages. Engineers as a whole end up taking a larger share of the revenue compared to VCs, who still simply just get their return r on c.

Proposition 5 summarizes the comparative static results from a positive shock to R.[23]

Proposition 5. *An increase in the size of the product market: (1) raises the wages of engineers, (2) raises the retained equity of entrepreneurs, (3) lowers the startup probability of success, (4) raises expected profits, (5) raises realized profits, and (6) increases the fraction of engineers pursuing entrepreneurship.*

Proof. See Appendix A.10. □

4 Social efficiency

By assumption, an idea only needs one entrepreneur to pursue it for society to reap the full social benefit of commercializing that idea. As such, one entrepreneur per idea is the efficient

[23] When the product market expands, startups become *smaller*, as engineer employees became expensive. While this perhaps seems counterintuitive, the phenomenon of startups with very high valuations but small headcounts is much remarked upon in Silicon Valley. For example, when WhatsApp was acquired by Facebook for $14 billion, it had 42 employees. Presumably, this remarkable outcome is related to the productivity of elite engineers, but it also suggests WhatsApp was economizing on expensive engineer labor.

allocation. However, subject to the constraint that engineers are free to choose their ideas and occupations, is the allocation of engineers between occupations at the constrained optimum? In other words, would a social planner want to adjust the equilibrium g? The answer depends on what the social planner is trying to maximize. Following Kihlstrom and Laffont (1979), I assume that the planner's goal is to maximize total revenue, minus non-sunk costs.[24] Given the cumulative nature of innovation, one could argue for weighting the first unit of some new good more heavily that than the last unit produced of some existing good.

The entrepreneurial system has $q_0 E$ successful startups, each of which generates $R\phi(l^*)$ in revenue. The social cost of generating this revenue is just the total startup costs, EC. Proposition 6 shows that the net social benefit is equal to the total number of engineers times the wage of employees, or

$$Sw = q_0 ER\phi(l^*) - EC.$$

Proposition 6. *The net social benefit of the entrepreneurial system is Sw.*

Proof. See Appendix A.11. $\square$

Now I turn to the question of whether the Sw generated by the system is as high as possible. If the decentralized allocation of engineers is efficient, then moving a small number of engineers from employment to entrepreneurship (or vice versa) should keep the net social benefit the same. Consider making $\mathrm{d}E$ employees become entrepreneurs. The social cost of removing $\mathrm{d}E$ engineers from employment is simply $w\,\mathrm{d}E$. The social benefit from this move depends on how many of the $\mathrm{d}E$ new entrepreneurs found startups pursuing otherwise un-pursued ideas.

The number of new ideas pursued by the $\mathrm{d}E$ entrepreneurs is $\kappa f(q_0)|\,\mathrm{d}q_0|$, each of which has a success probability of q_0. Let x be the probability that an engineer forced into entrepreneur-

[24] Using revenue allows me to sidestep the difficulties in defining surplus, as defining consumer surplus has proven challenging for the kinds of goods produced by innovative entrepreneurship (Brynjolfsson et al., 2003; Goolsbee et al., 2006).

ship pursues an idea not previously explored, and so

$$x = f(q_0)\kappa \left| \frac{\partial q_0}{\partial E} \right|.$$

The number of entrepreneurs must satisfy $E = \kappa \int_{q_0}^{1} \frac{q}{q_0} f(q)\, \mathrm{d}q$, and so if we differentiate this identity by E, we have $1 = \kappa(-f(q_0) - E/(\kappa q_0))\frac{\partial q_0}{\partial E}$, which allows us to write x as

$$x = 1 - |\eta_E^{q_0}|,$$

showing that the probability that a new entrant is pursuing a novel idea is $x = 1 + \eta_E^{q_0}$.

If the new entrepreneur generates a successful firm, the social benefit is π, which is revenue minus the labor cost.[25] Regardless of whether or not he or she is successful, the social cost is C. The social planner is indifferent about the occupation of the marginal engineer only when

$$x\left(q_0\pi\right) - C = w,$$

which only holds when $x = 1$, implying that $\eta_E^{q_0} = 0$: a social planner would only be indifferent regarding the flow into or out of entrepreneurship when there is no entrepreneur crowding. For all $|\eta_E^{q_0}| > 0$, the social planner would prefer more employment rather than more entrepreneurship, which is Proposition 7.

Proposition 7. *The decentralized allocation of engineers to entrepreneurship and employment is efficient only when the startup success probability is completely inelastic with respect to the number of entrepreneurs.*

Proof. See Appendix A.12 □

Other models of entrepreneurial occupational choice consider whether the decentralized

[25] Note that while engineer labor is sunk, the social cost comes from pulling away those employee engineers from some other startup.

occupational choices of individuals is efficient. For example, Kihlstrom and Laffont (1979) show that the decentralized allocation is efficient in their model when entrepreneurs are risk neutral. As I assume risk neutrality, I get the same result that the equilibrium is efficient when I assume there no idea competition, as in Khilstrom and Laffont.[26]

4.1 What if bad things happen to startups pursuing good ideas?

Startups might fail even if the business idea is viable. For this reason, it might be socially beneficial to have more than one startup pursue each idea. Further, it might be better to have more entrepreneurs pursue better ideas. The decentralized allocation, $n(q)$, has both of these features—with q/q_0 entrepreneurs per idea, the number of entrants per idea is increasing in idea quality. The natural question, however, is whether this is the allocation the social planner would pursue.

Let us give the social planner a pure allocation problem, fixing both g and q_0. The social planner has E entrepreneurs to distribute. Let the social value of a success in the product market be normalized to 1. Let $n^*(q)$ be the optimal number of entrepreneurs for an idea of quality q. Let us further require that the worst idea has to receive a unit mass of entrepreneurs, i.e., that $n^*(q_0) = 1$, just as $n(q_0) = 1$.

Now I introduce the possibility that a startup can fail for idiosyncratic reasons. Let $P(n)$ be the probability that at least one startup succeeds when n startups pursue an idea. The social planner's optimization problem is

$$\underset{n(\cdot)}{\arg\max} \quad \kappa \int_{q_0}^{1} q P(n(q)) f(q) \,\mathrm{d}q \quad \text{s.t.} \int_{q_0}^{1} f(q) n(q) \,\mathrm{d}q = E \text{ and } n(q_0) = 1.$$

Despite being a calculus of variations problem, the solution is simple: for all q, $qP'(n^*(q)) = k_0$, where k_0 is some constant: the social planner allocates engineers to ideas until the marginal re-

[26] Philippon (2010) considers the allocation between entrepreneurship, employment in the firms created by entrepreneurs and finance, finding that there is no efficiency reason to tax different sectors differently.

turns to another entrepreneur are the same across ideas. This solution depends on the shape of $P(\cdot)$. If $P(\cdot)$ were close to linear (which is unrealistic given that we expect declining marginal returns), then $n^*(\cdot)$ would tend to put most of the weight on the very best idea. The more concave $P(\cdot)$, the flatter $n^*(q)$ is in q.

One natural characterization of $P(\cdot)$ arises from assuming that individual startup success probabilities are independent of each other. Let that individual startup probability of success be p. If n startups pursue some idea, the probability that at least one succeeds is $P = 1-(1-p)^n \approx 1-\exp(-pn)$. The optimal $n^*(\cdot)$ in this case is

$$n^*(q) = k_1 - \frac{1}{p}\log\frac{k_1}{q},$$

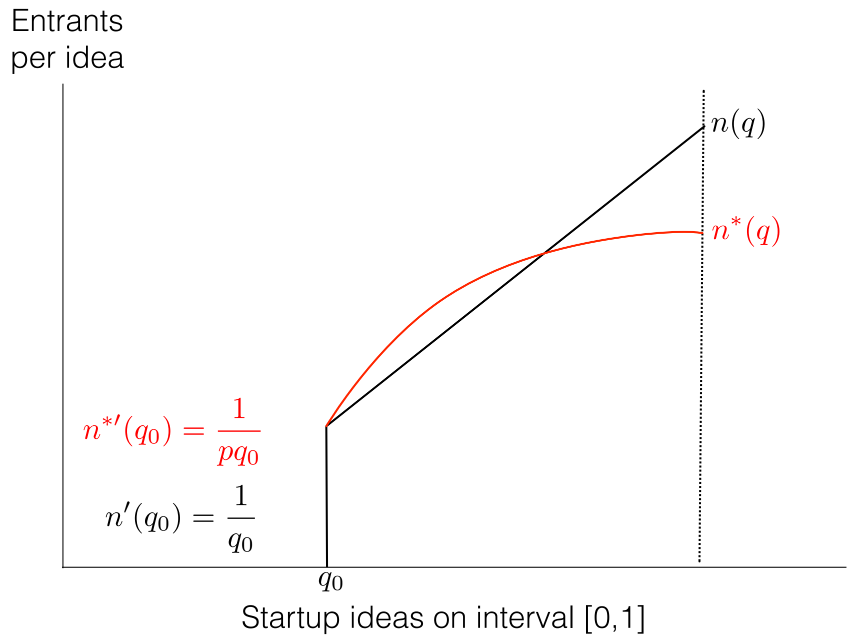
where k_1 and k_0 are constants that satisfy the summing up constraint and the boundary condition. Note that for all q, $n^{*\prime}(q) > 0$ and that $n^{*\prime\prime}(q) = 1/(pq^2) < 0$, i.e., that the optimal allocation is increasing, but concave, in q. Figure 7 shows the centralized allocation of engineers to ideas versus the optimal allocation. The optimal allocation has the same number of entrepreneurs at q_0 but has a steeper slope. As both allocation functions are monotonically increasing and have to have the same area under the curve, they eventually must cross. At that crossing point, all ideas to the right get too many entrepreneurs under decentralized allocation, relative to the optimal.

Proposition 8 formalizes this argument that the decentralized allocation gets too much entry on the "best" ideas.

Proposition 8. *If individual startup success probabilities are independent and identically distributed, then the optimal allocation would have more entrepreneurs pursue relatively lower quality ideas, under the approximation that* $1-(1-p)^n \approx 1-\exp(-pn)$*, where* n *is the number of engineers per idea and* p *is the individual startup success probability.*

Proof. See Appendix A.13. □

Figure 7: Optimal allocation versus decentralized allocation of entrepreneurs to ideas



Notes: This figure compares the decentralized allocation of engineers to ideas, q/q_0, to the optimal allocation under the assumption that each startup has some idiosyncratic risk of failure, $1-p$.

By including the possibility of startup failure, the stark inefficiency that arises in the decentralized allocation in the basic model is lessened: some of the "excess" entry on better ideas is actually desirable. However, Proposition 8 shows that even setting aside the allocative efficiency question of entrepreneurship versus ideas, the allocation of engineers has too much entry on relatively "sure" things.

The analysis of startup system efficiency presented here has been entirely theoretical, but there has been some empirical work focused on assessing the efficiency of the entrepreneurial "system" by examining trends in attributes over time. Two attributes that have received particular attention are (1) the number of new ventures formed and (2) their probability of "success" (variously defined as obtaining venture backing, having an IPO, gaining market share, and so on). Some of the trends in US entrepreneurship are worrying, with studies showing large declines in business dynamism (Decker et al., 2014, 2015; Haltiwanger et al., 2015). An empirical challenge in this work is that the underlying sources of data make it difficult to separate "conventional" entrepreneurship from the more consequential kinds. While no precise definition of consequential entrepreneurship exists, Fazio et al. (2016) highlight common traits, such as incorporation, non-eponymous naming, venture backing, patent applications, and so on.[27]

Focusing solely on consequential—or what they call "innovation-driven"—entrepreneurship, Fazio et al. (2016) find substantial growth in startup formation beginning in about 2010, but they also find evidence of a decline in the startup success rate, which one commentator succinctly summarized as "the US may have as many would-be Bezoses as ever, but it's getting fewer Amazons."[28]

A useful feature of the model presented in this paper is that it gives an interpretation to empirical results that Fazio et al. (2016) present. A lower success rate seems like bad news—and it is literally bad news for the individual entrepreneur—but the "social" or "system" interpretation of the decline is not so straightforward. Perhaps a technological innovation lowered startup costs, drawing more individuals into entrepreneurship, which in turn made the marginal funded startup less likely to succeed (Proposition 2). In this scenario, a lower success rate is probably the "right" outcome; an environment in which venture capitalists only fund "sure things" is unlikely to be socially efficient, especially when attempts have gotten cheaper. In contrast, it would be troubling if new startups were failing at a higher rate because there were simply fewer good business ideas worth pursuing (Proposition 4), ala Gordon (2016). The different causes of the decline should spur different policy responses.

[27] For the significance of eponymous naming, see Belenzon et al. (2014).

[28] Source: Ben Casselman, FiveThirtyEight Blog, http://fivethirtyeight.com/features/the-next-amazon-or-apple-or-ge-is-probably-failing-right-now/. Accessed Online: March 3rd, 2016.

5 Conclusion

This paper develops a model of innovative entrepreneurial clusters, focusing on the role of idea competition. In addition to presenting a number of policy-relevant comparative statics, the model also characterizes the efficiency of the entrepreneurial system. The degree of inefficiency depends on the elasticity of startup success with respect to the number of entrepreneurs. The model also highlights a way to make this startup success probability less elastic (and hence the system more efficient)—namely by expanding the space of potential startup ideas. Although the notion that investment in basic R&D can be socially beneficial is not a novel insight, the reason given in the model—to reduce entrepreneurial crowding—is, as far as I am aware, novel.

A natural direction for future work on the theory side would be to develop a dynamic version of the model. It would be particularly interesting to explore how systematic errors in estimating R, the extent of the product market, play out over time. If, for example, entrepreneurs believe R is higher than it is, there would be excess entry into entrepreneurship initially, but then slack labor demand once the disappointing "news" about R becomes known. On the other hand, a better-than-expected R could choke off the next flurry of startups by making the opportunity cost of entrepreneurship too high.[29] This dynamic analysis could be particularly useful, given the apparent tendency of some sectors to boom-and-bust dynamics.

On the empirical side, the obvious direction is to test the model predictions. A strength of the paper is that nearly everything of interest is either measured or is measurable. The technology sector is fast moving and subject to many shocks, making quasi-experimental research designs particularly credible. One important and obvious direction for future work would be to use the data available on which startups are competitors with each other—if combined with outcome data on which firm "won" the product market—to add more nuance to the simplified presentation of idea competition in the model.

[29] There is some evidence from Agrawal et al. (2015) on the importance of having large chunks of time to get some new side project started—something that unemployment provides but a high-intensity engineering role does not.

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A Proofs

A.1 Derivation of the elasticity of startup success probability with respect to ideas, from Section 3.1.1

Using Equation 1, following a small increase in κ, but keeping the number of entrepreneurs fixed, we can write

$$0 = \frac{E}{k} + \left(\kappa f(q_0) + \kappa \frac{E}{q_0}\right) \frac{\partial q_0}{\partial \kappa},$$

and as an elasticity,

$$\eta_{\kappa}^{q_0} = \frac{1}{1 + \kappa f(q_0) q_0 E^{-1}}.$$

A.2 Profits are always positive, from Section 3.2

Because $\phi(l)$ is concave,

$$\phi(l^*) = \int_0^{l^*} \phi'(x)dx > l^* \phi'(l^*),$$

and so $R\left(\phi(l^*) - \phi'(l^*)l^*\right) = \pi > 0$ (recall that employees are paid their marginal product).

A.3 Derivation of Equation 8

The demand curve is $D = Eq_0 d(w)$. Differentiating with respect to the number of entrepreneurs gives

$$\begin{aligned}
\frac{\partial D}{\partial E} &= q_0 d(w) + Ed(w)\frac{\partial q_0}{\partial E} \\
\frac{1}{D}\frac{\partial D}{\partial E} &= \frac{1}{E} + \frac{1}{q_0}\frac{\partial q_0}{\partial E} \\
\frac{E}{D}\frac{\partial D}{\partial E} &= 1 + \frac{E}{q_0}\frac{\partial q_0}{\partial E} \\
\eta_E^D &= 1 + \eta_E^{q_0}.
\end{aligned}$$

A.4 Proof of Proposition 1

Proof. The plan of the proof is to (1) show that the returns to employment and entrepreneurship move in opposite directions with respect to g, the fraction of engineers choosing entrepreneurship, and (2) show that as nearly everyone pursues employment, or $g \to 0$, the returns to entrepreneurship are greater than the returns to employment, and vice-versa when nearly everyone pursues entrepreneurship, $g \to 1$. This ensures that two return curves cross in the unit interval, i.e., that there exists some equilibrium $g \in (0,1)$. With this g, the returns to employment and entrepreneurship are equalized, which is required for an equilibrium. As the return curves are monotonic, this equilibrium is unique.

Opposite signs for the slopes of w and g. A larger g means more entrepreneurs and hence more successful firms demanding labor; it also means fewer employees in the labor market. The higher the demand and the lower the supply, the lower wages are and so $w'(g) > 0$. More entrepreneurs reduces the returns to entrepreneurship. With more entrepreneurs, q_0 goes down (by Equation 2). Because of the higher wages for employees at this higher g, profits go down. Finally, e goes down, as lower expected profits means that entrepreneurs must give up more equity to obtain seed funding. As such, $y'(g) < 0$.

Returns cross for some g in $(0,1)$. Although the slopes of $w(g)$ and $y(g)$ have opposite signs, for an equilibrium to exist, they must cross in the unit interval, i.e., that there is a $g \in (0,1)$ such that $w(g) = y(g)$. First, consider the case when $g \to 0$, meaning almost every engineer is an employee. Let $\bar{q} \leq 1$ be the startup success probability when there are almost no entrepreneurs. By assumption, $C/R < \bar{q}$ (recall that the proposition states that the equilibrium exists only for a sufficiently small C/R). As the number of entrepreneurs gets arbitrarily small, the number of successful startups gets arbitrarily small, and thus the number of workers per firm approaches S. The returns to entrepreneurship minus the returns to employment is

$$\lim_{l \to S} \bar{q}\left(R\phi(l) - R\phi'(l)l\right) - C - R\phi'(l) = \bar{q}R - C.$$

Recall that by assumption, as $l \to S$, $\phi(l) \to 1$ and $\phi'(l) \to 0$. As $\bar{q}R - C > 0$, as the number of entrepreneurs gets arbitrarily small, the returns to entrepreneurship are greater than the returns to employment.

Now we consider the case when $g \to 1$, meaning that nearly all engineers pursue entrepreneurship. With almost no engineers available to be employees, $L \to 0$. The returns to entrepreneurship minus the returns to employment is

$$\lim_{L \to 0} \underline{q}\left(R\phi(l) - R\phi'(l)l\right) - C - R\phi'(l) = -C - Rp,$$

where $p = \lim_{l \to 0} \phi'(l)$. As $p > 0$, $C > 0$ and $R > 0$, the returns to entrepreneurship are lower than the returns to employment. We can now conclude that there is some $g \in (0,1)$ such that the returns to employment and entrepreneurship are equalized, or $w(g) = y(g)$. As $w(\cdot)$ and $y(\cdot)$ are monotonic, the equilibrium is unique. □

A.5 Proof of Lemmas

Two lemmas are useful for simplifying the comparative static prediction proofs. Lemma 1 shows that w and q_0 move in opposite directions for all shocks (except those to the supply of engineers and/or the supply of ideas). Lemma 2 relates the wages of engineers to changes in the supply of engineers and the supply of startup ideas.

Lemma 1. *Any small change in an exogenous variable (with respect to S and κ) that raises w decreases q_0 and vice versa.*

Proof. The labor market clearing condition $S = E + Eq_0 l^*$ can be written as

$$d(w) = \frac{S}{\kappa} \frac{1}{\int_{q_0}^{1} q f(q) dq} - \frac{1}{q_0}.$$

Let γ be some exogenous variable that does not affect κ or S but does affect w and q_0. If we differentiate the market clearing condition by γ, we have

$$d'(w)\frac{\partial w}{\partial \gamma} = \frac{\partial q_0}{\partial \gamma}\left(1 + \frac{S}{\kappa}\frac{f(q_0)q_0^3}{\left(\int_{q_0}^1 qf(q)dq\right)^2}\right)\frac{1}{q_0^2}.$$

On the right-hand side, the last two factors are both positive. And since $d'(w) < 0$, $\frac{\partial w}{\partial \gamma}$ and $\frac{\partial q_0}{\partial \gamma}$ have opposite signs. □

Lemma 2. *Engineer wages are decreasing the ratio of engineers to ideas.*

Proof. Let the ratio of engineers to startup ideas be $a = S/\kappa$. Using the fact that $g = \frac{1}{1+q_0 d(w)}$, the market clearing condition can be written as

$$d(w) = a\frac{1}{\int_{q_0}^1 qf(q)dq} - \frac{1}{q_0}.$$

Differentiating with respect to a, we have

$$d'(w)\frac{\partial w}{\partial a} = \frac{1}{\int_{q_0}^1 qf(q)dq} + \frac{\partial q_0}{\partial a}\left(\frac{1}{q_o^2} + \frac{af(q_0)}{\left(\int_{q_0}^1 qf(q)dq\right)^2}\right).$$

Assume that $\frac{\partial w}{\partial a} > 0$. Since $d'(w) < 0$, the entire right-hand side must be negative, which implies that $\frac{\partial q_0}{\partial a} < 0$. However, if we differentiate the expression for employee wages, Equation 5, by a, we get

$$\frac{\partial w}{\partial a}\left(1 + d(w)q_0\right) = \pi\frac{\partial q_0}{\partial a}, \tag{22}$$

which implies that $\frac{\partial w}{\partial a}$ and $\frac{\partial q_0}{\partial a}$ have the same sign, contrary to our original assumption. Therefore, $\frac{\partial w}{\partial a} < 0$. □

A.6 Proof of Proposition 2

Proof. For (1), starting from $C = q_0\pi - w$, and differentiating by C, we have

$$1 = \pi \frac{\partial q_0}{\partial C} - \left[1 + q_0 d(w)\right] \frac{\partial w}{\partial C}. \tag{23}$$

Assume that $\frac{\partial q_0}{\partial C} < 0$. For Equation 23 to hold, it would imply that $\frac{\partial w}{\partial C} < 0$, but this is a contradiction by Lemma 1, and therefore $\frac{\partial q_0}{\partial C} > 0$, which is claim (3). This implies that $\frac{\partial w}{\partial C} < 0$ by Lemma 1, which is claim (1). For claim (2), as $e = w/(w+c)$, we know that retained equity falls when startup costs increase. For claim (4), since q_0 goes up and w goes down, expected profits, $q_0\pi$, go up. For claim (5), as w is lower, realized profits go up as well. For claim (6), since w goes down, $d(w)$ goes up. As we know from claim (3), q_0 goes up, and therefore by Equation 6, entrepreneurship goes down. □

A.7 Derivation of supply shock effects from Section 3.9

Before any adjustment in occupations, market clearing in the labor market requires that

$$\begin{aligned}
dw &= \frac{-(1-g)\,\mathrm{d}S + \frac{\partial D}{\partial E} g\,\mathrm{d}S}{|\partial D/\partial w|} \\
\frac{S}{w}\frac{dw}{dS} &= \frac{-S(1-g) + \frac{\partial D}{\partial E} gS}{w|\partial D/\partial w|} \\
\eta_S^w &= \frac{-1 + \eta_E^D}{|\eta_w^D|} \\
\eta_S^w &= \frac{-1 + 1 + \eta_E^{q_0}}{|\eta_w^D|} \\
\eta_S^w &= \frac{\eta_E^{q_0}}{|\eta_w^D|}.
\end{aligned}$$

A.8 Proof of Proposition 3

Proof. The lowering of wages, which is claim (1), follows from Lemma 2, as $\frac{\partial a}{\partial S} > 0$ (recall that $a = S/\kappa$). For claim (2), from the fact that $e = w/(w+c)$, we know that retained equity falls as well since $\frac{\partial}{\partial w}\left[\frac{w}{w+C}\right] > 0$. For claim (3), from Equation 5, $\frac{\partial w}{\partial S} = \frac{\partial}{\partial S} q_0 \pi < 0$ (making use of the fact that wages fall). For claim (4), $\frac{\partial \pi}{\partial S} = -d(w)\frac{\partial w}{\partial S} > 0$, as $d(w) > 0$ (and again making use of (1)). For claim (5), from the occupational indifference condition, we know that

$$\frac{\partial w}{\partial S} = \frac{\partial}{\partial S}[q_o \pi],$$

and from the envelope theorem, $\frac{\partial \pi}{\partial S} = -d(w)\frac{\partial w}{\partial S}$, and so

$$\frac{\partial w}{\partial S}\big(1 + q_0 d(w)\big) = \pi \frac{\partial q_0}{\partial S},$$

and since $\frac{\partial w}{\partial S} < 0$, $\frac{\partial q_0}{\partial S} < 0$. For claim (6), as w goes down, $d(w)$ goes up, but as q_0 goes down, the overall effect on the entrepreneurship fraction, g is ambiguous. □

A.9 Proof of Proposition 4

Proof. For claim (1), when κ increases, the ratio of engineers to ideas decreases ($a = S/\kappa$), and so it directly follows from Lemma 2 that engineer wages increase. For claim (2), as startup costs have not changed, because $e = w/(w+C)$, it follows that retained equity increases. For claim (3), expected profits rise since wages rise, i.e., $\frac{\partial w}{\partial \kappa} = \frac{\partial}{\partial \kappa}[q_0 \pi] > 0$. However, for claim (4), realized profits fall because wages increase. For claim (5), recall that

$$\frac{\partial w}{\partial a}\big(1 + d(w) q_0\big) = \pi \frac{\partial q_0}{\partial a}, \tag{24}$$

which implies that $\frac{\partial w}{\partial a}$ and $\frac{\partial q_0}{\partial a}$ have the same sign. From claim (1), we know that $\frac{\partial w}{\partial a} < 0$, and hence that $\frac{\partial q_0}{\partial a} < 0$. Therefore $\frac{\partial q_0}{\partial \kappa} > 0$, meaning that the startup probability of success increases. For claim (6), because w goes up, $d(w)$ goes down, but q_0 goes up. As such, the effect of a change in the stock of ideas on $q_0 d(w)$ is ambiguous, and hence the effect on entrepreneurship is ambiguous. □

A.10 Proof of Proposition 5

Proof. For claim (1), differentiating the expression for engineer wages, Equation 5, by R, we have

$$\frac{\partial w}{\partial R} = q_0 \phi(l) + \pi \frac{\partial q_0}{\partial R}. \tag{25}$$

Assume that $\frac{\partial w}{\partial R} < 0$. As $q_0 \phi(l) > 0$, for the equation above to hold, $\pi \frac{\partial q_0}{\partial R} < 0$, but by lemma 1, $\frac{\partial q_0}{\partial R} > 0$, given the assumption that $\frac{\partial w}{\partial R} < 0$. As startup costs have not changed, claim (2) follows directly from the condition that $e = w/(w + C)$ and claim (1). For claim (3), because wages rise, q_0 falls. For claim (4) expected profits rise, as $\frac{\partial w}{\partial R} = \frac{\partial}{\partial R}[q_0 \pi]$. And for claim (5), the effect of a larger product market on realized profits is also positive, as expected profits rose despite q_0 falling. For claim (6), as both q_0 and $d(w)$ go down, g goes up. □

A.11 Proof of Proposition 6

Proof. The net social benefit (total revenue minus startup costs) from the decentralized allocation of engineers is

$$
\begin{aligned}
&= q_0 gS[R\phi(l^*)] - gSC \\
&= gS\left(q_0[R\phi(l^*)] - C\right) \\
&= gS\left(q_0[\pi + wl^*]] - C\right) \\
&= gS\left(q_0[\pi + wl^*]] - \pi q_0 + w\right) \\
&= gS\left(w\left[1 + q_0 l^*\right]\right) \\
&= gS\left(\frac{w}{g}\right) \\
&= Sw.
\end{aligned}
$$

□

A.12 Proof of Proposition 7

Proof. For ease of exposition, let $\mathrm{d}E = 1$, i.e., a single engineer moves to entrepreneurship. This has a direct cost of C. If the marginal engineer does happen to choose a "new" idea to pursue and it is successful, then there is $R\phi(l)$ more output. However, producing this output costs wl^*, as engineers must be drawn from other successful firms. The marginal engineer offers the same social benefit in entrepreneurship as in employment if

$$\left(1-|\eta_E^{q_0}|\right)(q_0\pi)-C=w$$
$$\left(1-|\eta_E^{q_0}|\right)(q_0\pi)=w+C$$
$$\left(1-|\eta_E^{q_0}|\right)(q_0\pi)=q_0\pi$$
$$1-|\eta_E^{q_0}|=1$$

or when the startup success probability with respect to the number of entrepreneurs is completely inelastic, $|\eta_E^{q_0}|=0$. □

A.13 Proof of Proposition 8

Proof. By assumption, $n^*(q_0)=n(q_0)$, and since $p<1$, $n^{*\prime}(q_0)>n(q_0)$. There exists a q' such that for all $q\in(q_0,q')$, $n^*(q)>n(q)$. As the area under $n(q)$ must equal the area under $n^*(q)$, the two curves must eventually cross. And because both have the same value at q_0 and are monotonically increasing (but do not have the same slopes everywhere), they cross only once. As $n(q)$ has a constant slope, and $n^*(q)$ has a greater slope at q_0, $n^*(q)$ crosses from above. Let $\hat{q}$ be that crossing point. As $\int_{q_0}^{\hat{q}} n^*(q)f(q)\,\mathrm{d}q>\int_{q_0}^{\hat{q}} n^*(q)f(q)\,\mathrm{d}q$, and since the area under the two curves must be equal, the optimal curve $n^*(q)$ is everywhere below the $n(q)$ curve for $q>\hat{q}$. □